ELECTRIC VEHICLE CHARGING STATION PLACEMENT AND MANAGEMENT

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INTERDISCIPLINARY GRADUATE SCHOOL

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The Joint NTU-UBC Research Centre of Excellence in Active Living for the Elderly (LILY)

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Abstract

Due to the world's shortage of fossil fuels and the serious environmental pollution from burning them, seeking alternative energy has become a crucial topic of research. Transportation is one of the main consumers of energy and contributors to air pollution. Electric Vehicles (EVs) move pollution away from urban areas and electricity can be efficiently transformed from both traditional fossil fuels and promising renewable energies like solar energy and tidal energy. EVs, as a replacement of traditional internal combustion engine vehicles, provide an environment-friendly solution to modern cities' transportation. A rapid growth of EVs has been seen in recent years along with the rising popularity of the notion of smart cities. This calls for an efficient deployment of relevant supporting facilities, among which charging facility is of top priority. Although EVs can be charged at home, it is time-consuming and usually takes 6 to 8 hours, which is at least 12 times the time it takes at charging stations with high voltage. The distribution of charging stations determines EV drivers' accessibility to energy sources and consequently affects the EV flow and traffic conditions in the road network. Although charging in charging stations is much faster than that with domestic electricity, it can still take several dozens of minutes. Thus in return, the EV drivers' charging behavior would greatly influence the performance of the charging system, especially the queuing condition in charging stations. This thesis is concerned with optimal placement and efficient management of charging stations. To achieve this goal, we carefully study the interactions between charging stations and EV drivers as well as the bounded rationality of EV drivers in charging activities.

In our first step of research, we study the electric vehicle charging station placement problem. We highlight two main factors to consider: traffic congestion and charging

station congestion. We also take into consideration the electric vehicle drivers' strategic charging activities. A congestion game framework is employed in our work to model the electric vehicle drivers' competitive and self-interested charging activities. We formulate the charging station placement problem as a bi-level optimization problem and propose efficient algorithms for computing optimal solutions. Experimental results show that our approach provides a better result than baseline methods.

We then extend to optimal pricing for charging station management. While most existing research works focus on optimizing spatial placement of charging stations, they are inflexible and inefficient against rapidly changing urban structure and traffic pattern. Therefore, this work approaches the management of EV charging stations from the pricing perspective as a more flexible and adaptive complement to established charging station placement. In this work, we build a realistic pricing model in consideration of residential travel pattern and EV drivers' self-interested charging behavior, traffic congestion, and operating expense of charging stations. We formulate the pricing problem as a mixed integer non-convex optimization problem and propose a scalable algorithm to solve it. Experiments on both mock and real data are conducted, which show scalability of our algorithm as well as our solution's significant improvement over existing approaches.

Last, we study charging behavior of the EV drivers and construct more practical charging behavior models. While previous works assume that EV drivers can reach equilibrium in the charging game, this can rarely happen in real world. Players are limited by partial information and poor computation ability, thus they are bounded rational. Through analyzing EV drivers' decision-making in the charging process, we propose a 2-Level Nested LQRE charging behavior model that combines LQRE model and level-k thinking model. We design a set of user studies to simulate the charging scenarios, collect data from human players and learn parameters of the 2-Level Nested LQRE charging behavior model. Experimental results show that our charging behavior model well captures the bounded rationality of human players in the charging process. The selection distribution of all players tends to converge after a number of repeated playing. Furthermore, we formulate the charging station placement problem with the 2-Level Nested LQRE model and design a heuristic algorithm to solve it. Our approach obtains placement with a significantly better performance by decreasing more than 8% for the social cost compared with benchmark approaches.

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Chapter 1

Introduction

1.1 Charging Station Placement and Management

Electric vehicles (EVs) have revealed great interest from the public, the government and the academy in recent years. The popularity of EVs is contributed by their outstanding advantages comparing to traditional inner-combustion vehicles including (1) comfort, (2) economy and (3) cleanness. EVs use electricity to power the vehicle, thus they are more stable and there would be no smell of fossil fuels. The cost of electricity is lower than that of fossil fuels, thus EV is more economical. Besides, fossil fuels are limited resources and it takes too long for them to reborn. Electricity can be efficiently transferred from other forms of energies, like solar energy, tide energy, wind energy and even nuclear energy. Despite that, electricity is more environmental-friendly. The consumption of fossil fuels by vehicles has caused serious air pollution, especially in urban cities. Switching to electric vehicles would greatly release this issue as the production from energy consumption would turn to water rather than SO_2 from burning fossil fuels. With above-mentioned advantages, the society has made great effort to research on and develop EVs in the past years.

Despite the techniques to construct the EVs, another vital element for promoting EVs is the supporting facilities. Due to the limited battery capacity and mileage (usually around $200km$), EVs need recharging frequently, as traditional inner-combustion vehicles need refueling. According to the report, the mileage anxiety is possible to hinder the diffusion of EVs. Moreover, recharging an EV takes much more time than refueling one vehicle. While refueling usually takes a few minutes (taking normal size car for example), recharging can take as much as 12 times of that in charging stations with specially supplied voltage. Although EV users can also charge their vehicles with domestic electricity, it would take even more time (6 to 8 hours). Besides, not every citizen is equipped with his/her own garage to charge an EV at home. As a result, the supporting facilities, especially electric vehicle charging stations are of essential need for EV drivers.

The distribution and size of electric vehicle charging stations directly decide the accessibility and convenience of EV drivers. While constructing charging stations, besides the hardware conditions like geography limitations and the software factors like economic budget, there is another momentous element to consider, i.e., the interaction between the charging stations and the EV drivers. At first, the EV drivers select charging stations considering their positions and serviceability. Intuitively, EV drivers would prefer charging stations that are easier to access from their start points (i.e., from where they will go for charging). The charging fare in charging stations is also influential in their decision. Another significant factor is the serviceability of the charging station, which would affect the queuing condition of EV drivers. As we mentioned in the previous paragraph, although the charging process in charging stations is shorter than that at home, it can still take dozens of minutes. Thus, the queues in charging stations can result in long waiting time, which can hinder the adoption of EVs [1, 2]. In return, the charging decisions of EV drivers would have an impact on the service quality of charging stations, and even to the traffic network when the proportion of EVs in the traffic network overwhelmingly exceed the proportion of other vehicles. As we can see, the interaction between the charging stations and EV drivers has a great impact on the performance of the whole charging system.

At the same time, EV drivers are competing with each other for the limited resources in the charging process. The charging stations, as well as the traffic network, are congestible resource, which means the cost of usage on the resources (i.e., queuing time or driving time) would increase with the number of EV drivers that use them. Thus the interactions among the EV drivers are also of our interest since they can decide the performance of the charging system and traffic network.

Such complex properties of charging stations make it an interesting but challenging research topic to optimally place the charging stations. Furthermore, charging station placement is irreversible due to the high cost of construction. Meanwhile, the population distribution, which directly decides the charging demand from the EV owners, would change greatly in the near future. In this case, the demand and supply relationship would change even the planning of charging stations has been carefully designed at the beginning.

With the development of cities and the urban structure changes, the charging station system needs additional methods to affect EV drivers' charging behavior and adjust the service quality of charging station system. Compared with removing and placing new charging stations, pricing is easily and immediately implementable without additional cost or waste of resources. Dynamic pricing schemes adapt to either long-term changes of travel demand caused by residential movements or short-term variances between peak and non-peak time and serve as a flexible complement to existing charging station placement.

To adapt to the urban structure change as well as varying charging demand, a practical solution as we propose in this thesis is to leverage the charging price to readjust EV users' charging behavior and improve the efficiency of the charging network. Compared with placement, pricing is easily and immediately implementable without additional cost or waste of resources. Dynamic pricing schemes adapt to either long-term changes of travel demand caused by residential movements or short-term variances between peak and non-peak time and serve as a flexible complement to existing charging station placement. Our goal is to optimize the pricing scheme to optimize the efficiency of charging stations, i.e., to minimize the additional cost caused by EV users' charging behavior, which is referred as the social cost. There have been some works leveraging dynamic pricing to improve the efficiency of public transportation systems, such as taxi systems [3, 4]. Some works have particularly focused on real-time pricing and charge-discharge policy for EV management [5, 6]. However, their aim is merely to balance electricity load in power grids, while traffic condition is not in their consideration.

Moreover, their method cannot be incorporated with trivial modifications because the traffic condition deeply relates to EV users' self-interested charging behavior associated with a graph-based road network, which is all absent from existing work.

In the process of charging station placement and pricing, the interaction between charging stations and EV drivers is always playing an important role as we discussed in above paragraphs. While there have been a number of works [7–10] on Charging Station Placement Problems (CSPP), only a few of them [7, 10] consider EV drivers' charging behavior in their works. The same happens on related works of charging pricing [5, 6], which mainly focus on balancing the electricity burden and power network supply. Moreover, among the few works that take into consideration EV drivers' charging behavior, their behavior models are based on simplistic assumptions. First, existing charging behavior models are lack of comprehensive study of EV drivers' preference over different factors in their charging process; second, they assume that EV drivers are fully rational more follow existing models without careful study and proof.

1.2 Problem Statement

Named as "Garden City", Singapore has a good reputation for its nice environment and air quality. However, there is still pollution around here, especially the air pollution caused by the heavy traffic that surrounds us all day. According to the official data [11], 20% of the total carbon emission and 75% of the air pollution in Singapore are caused by the land transportation system, mostly attributed by the motorized traffic. As a result, Singapore government is working on mitigating the environment problem due to the traffic system by introducing the clean EVs as the replacement for traditional internal combustion vehicles. The Singapore authorities have started to test the possibility and feasibility of introducing EVs into Singapore since 2011. As a metropolis with advanced energy network, the electric-car manufacturer BYD Asia-Pacific announced that Singapore has the "best potential" to implement EVs [12].

The construction of EV charging stations is the first challenge, to which the government needs to rise for the successful introduction of EVs. Besides the finance concern, there are some elements far more important and urgent, among which traffic condition is of top priority. The planning of charging stations calls for a careful investigation to avoid aggravating the traffic congestion of this small city. Specifically, Singapore is a small metropolis with a very small territory of $718.3km^2$ only (Fig. 1.1). The maximum east-west distance is $42km$, while the north-south distance is barely $23km$. In a city like Singapore, the most commonly considered problem, namely the limited EVs mileage (usually above $100km$ and some can exceed $500km$ [13]) is not a big issue. Contrasting with the small territory of Singapore is its huge population, which also means a large population of vehicles. According to the Singapore official announcement, there are more than $970,000$ motor vehicles in the year 2014 on this small island, which indicates the heavy traffic. The fact implies that rather than limited mileage, we should make more efforts on balancing the traffic in consideration of EV users' charging behaviors.

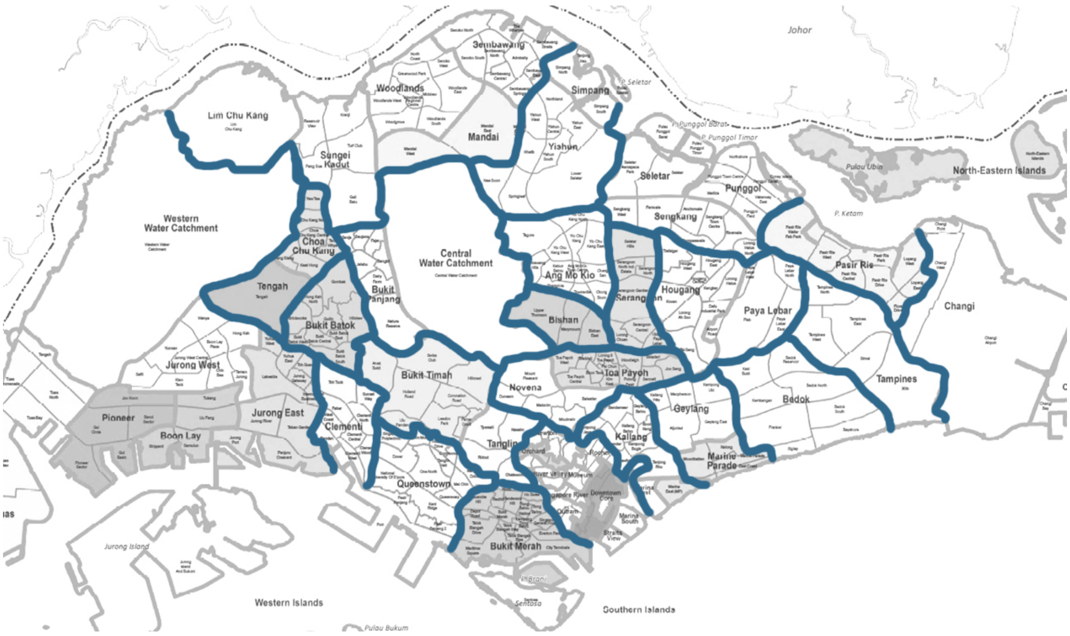


FIGURE 1.1: Zonal map of Singapore

As a highly developed metropolis with open attitudes toward cutting-edge technologies, yet a country with limited natural resource and energy supply, Singapore is actively seeking the possibility of mass adoption of EVs to support its sustainable development. Ever since 2011, its authorities have started an EV test-bed to study the feasibility of EVs on its road. More recently, in the Government's sustainable blueprint to guide the country's development over the next 15 years launched in 2014, Singapore has even

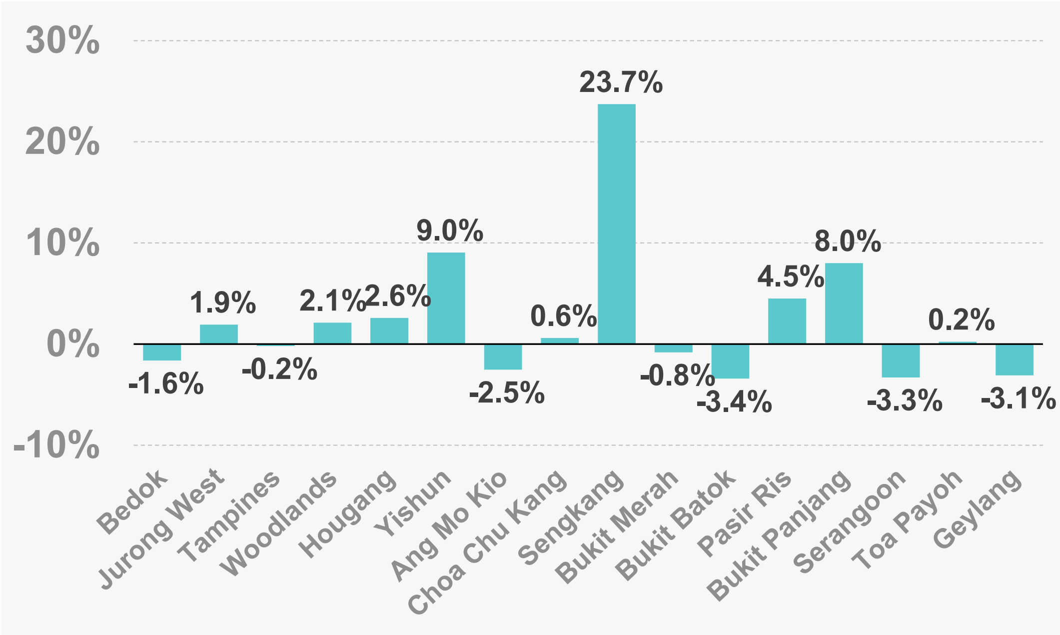


FIGURE 1.2: Population growth rate of major residential zones of Singapore from 2010 to 2015

planned to lead an EV-sharing project to make the new technology even more convenient and environmentally-friendly.

Indeed, the relatively short driving distances on the small territory and the advanced power grid of Singapore make EVs a good option for this city. However, there are also many difficulties that require every step taken to be carefully planned. Because of the land scarcity and the fact that roads have already taken up 12 percent of Singapore's total land area, there is limited room for further expansion of Singapore's road network. This leaves Singapore a very high road density of $4.8km/km^2$ and a transportation system that is highly sensitive to any changes to the current transportation mode. Besides, Singapore is undergoing a rapid change in the residential pattern along with its continuing development. As shown in Figure 1.2, population growth varies significantly among major residential zones of Singapore, indicating similar significant changes in residential traffic pattern. A sustainable plan, therefore, needs to be compatible with the current system while adaptable to future changes, to ensure a smooth transition toward the new EV-led transportation mode.

The real-world case of Singapore city motivates our work and offers us a concrete study case. In this thesis, the presented three works are aiming at developing approaches

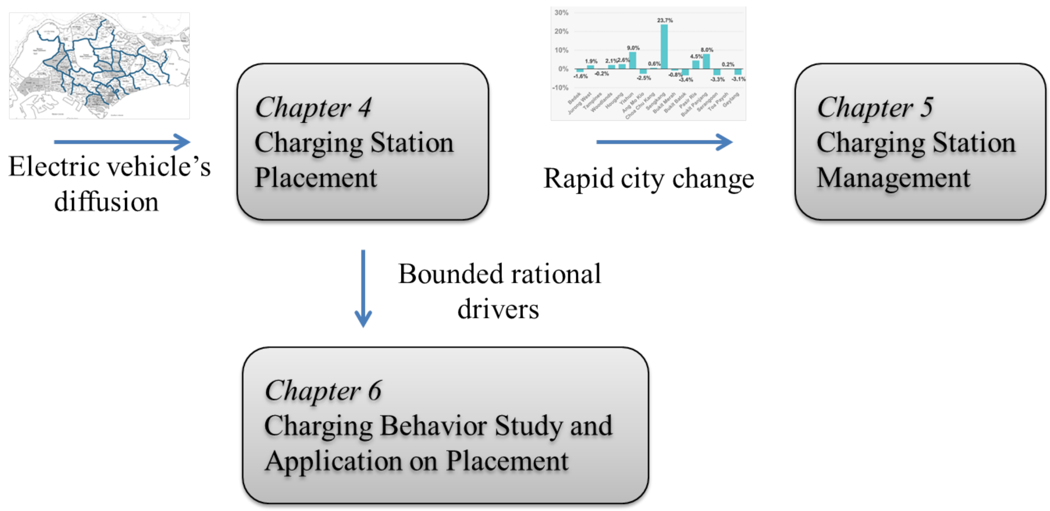


FIGURE 1.3: Three main works of the thesis

to solve electric vehicle charging station placement and management problem for cities like Singapore. The typical features of such small metropolis are fully taken into consideration in problem formulations. However, our approaches can easily be extended to other real-world problems. In each work, we provide such discussion while introducing the problem models.

1.3 Objectives and Contributions

We describe our objectives in dealing with unfair rating attacks, regarding all the problems mentioned in the section above. And also we present our main contributions on the way of achieving the objectives. The overall structure and relationship of the three main works in this thesis is shown in Figure 1.3.

For successful diffusion of electric vehicles, the support from charging stations is essential and vital. To find optimal charging station placement, we formulate a realistic model for the charging station placement problem in cities like Singapore considering the interactions among charging station placement, EV drivers' charging activities, traffic congestion and queuing condition in charging stations. We target in highly centralized and developed cities like Singapore, and thus the mileage anxiety of EV drivers is not considered. Through exploiting the structure of solution concept of the charging game, we transform an equivalent single-level charging station placement problem from the bi-level optimization formulation obtained. We develop an effective heuristic approach that can speed up the mixed integer single-level optimization problem with a large amount of non-linear constraints. We conduct theoretical analysis on PoA and corresponding experiments for the charging game. To validate the proposed approach, we demonstrate experimental results based on real data from Singapore, which show that our approach can provide an effective allocation of charging stations and outperforms baseline methods significantly.

Considering that charging station placement is not once for all, we conduct the continuous work based on the placement problem. The city population grows and shifts along with the unbalanced development in different parts of the urban area. To improve the performance of settled charging stations and make it adaptive to city changes, we suggest to incentivize the EV drivers' charging behavior with pricing method. To compute optimal pricing for charging station management, we take a game-theoretic perspective to study the EV charging station pricing problem motivated by the practical need of EV promotion in Singapore. The first point is a novel pricing model that comprehensively incorporates EV users' self-interested charging behavior and their various traffic patterns, traffic congestion contributed by EVs and other non-EV vehicles in the road network, as well as the financial concern for a sustainable operation of the charging network. The second point is the algorithm, *SSGA*, to solve the mixed integer non-convex optimal pricing problem, which features two key rules that guarantee efficient converging to equilibrium solution and drastically improves the running time performance. Last, our extensive experiments and results demonstrate our approach in several aspects, including solution quality, scalability, and robustness. Moreover, we compare our approach with uniform pricing and demonstrate how and to what extent *SSGA* can help with improving the traffic system efficiency and decreasing social cost caused by EV owners' charging behavior. Our approach can be applied in various modern cities like the motivating example Singapore to manage the charging stations in the future. We are actively approaching authorities of Singapore to look for such potential application.

Furthermore, we relax the strong assumption on EV drivers' fully rational charging behavior in the placement and management work. Considering the real-world scenarios, we can see that human drivers can hardly follow the equilibrium strategies due to their limited computation ability etc. Thus, we improve the placement of charging stations with the consideration of bounded rational human behavior to make the solution more robust for adoption on real-world problems. We formulate the EV Charging Station Placement problem (CSPP) with consideration of the bounded rational charging behavior of EV drivers. We propose a 2-Level Nested LQRE-Charging behavior model for the EV drivers to capture the EV drivers' irrational charging behaviors. From the human data and the analysis on it, we find that (1) human players rarely consider the influence from others' charging behavior and (2) they make decisions based on stationary factors. With the 2-Level Nested LQRE-Charging behavior model, we compute the optimal solution for CSPP and compare it with two benchmarks. The experimental results show that our approach significantly outperforms the benchmark in terms of the social cost, average queuing time and the maximum queuing time that the EV drivers would encounter. Our approach provides a better charging station placement, which can improve EV drivers' charging experience. This could be helpful in promoting EVs to the public. The EV charging behavior model can also be applied to other relating problems. For example, when charging stations have been constructed, governors can use pricing as a method to guide the EV drivers.

1.4 Thesis Organization

The first chapter of this thesis has introduced the background and motivation of the proposed research. Game theory will be employed to model and analyze the facility location and management problems. We also combine game theory with behavior study to understand the bounded rational charging behavior of EV drivers.

In the following, Chapter 2 reviews the prior research works on facility location, facility management (especially for dynamic pricing method), game theory (especially for congestion games, including the pure and mixed equilibrium computation as well as the price of anarchy and stability), optimization and bounded rational human behavior study. For the ease of understanding, relating theoretical knowledge and concepts, based on which we develop our approaches, are presented in Chapter 3.

Chapter 4 introduces the work on charging station placement. Firstly, we build a realistic Charging Station Placement Problem (CSPP) model, in which the EV drivers' strategic charging behaviors, the traffic condition and the queuing time in charging stations are considered. The overall objective is set as minimizing the total charging cost of EV drivers (named social cost), and EV drivers are assumed to minimize their charging cost with strategic charging behavior. We formulate the CSPP as a bi-level optimization problem, where we take the social cost as the upper-level objective, which is the goal of the government (who is assumed to be the one to decide the placement of charging stations); a charging game (which falls into the class of congestion games) is formulated in the sub-level problem and Nash Equilibrium is adopted to define the EV drivers' charging behaviors. Secondly, we successfully transfer the bi-level optimization problem into an equivalent single-level optimization problem by analyzing the definition and structure of the Nash Equilibrium in the charging game. We propose the algorithm OCEAN (Optimizing eleCtric vEhicle chArging statioN placement) to compute the optimal charging station placement. However, the real-world problems have a large scale of variables, and OCEAN is unable to handle them due to the existence of integer variables and the huge variable space. Thus we furthermore work out a heuristic algorithm OCEAN-C (OCEAN with Continuous variables) that can handle the real-world CSPP and ensure solution quality. Thirdly, we design and execute a lot of experiments for both mock data and the real situation of Singapore. The experimental results prove that the designed algorithms OCEAN-C can efficiently solve the CSPP and our approach outperforms some typical baseline methods.

Chapter 5 is the work on optimal pricing for charging station placement. We take a game-theoretic perspective and build the problem on a non-atomic congestion game played by EV users. The model incorporates the following key features: 1) EV users' self-interested charging behavior that they strategically choose the best charging plan (i.e., where to charge and how to reach the charge station) to minimize their costs including charging fees, traveling time, and queuing time; 2) EV users' traffic pattern with complex spatial variances; 3) Traffic congestion in the road network that is affected by both the EVs and other external vehicles; and 4) A budget constraint that ensures sufficient income to support the sustainable operation of the charging network. Using this model, we formulate the EV charging station pricing problem as a mixed integer non-convex optimization problem and propose a scalable algorithm to solve the problem, in particular, to deal with the large strategy space of the EVs. Experiments on both mock and real data are also conducted, which show scalability of our algorithm as well as our solution's significant improvement in social cost over existing approaches. A concrete instance is also used to visualize the difference between our approach and existing approaches.

Chapter 6 presents the charging behavior study and the optimization for charging station placement problem. The first contribution of this work is a realistic charging behavior model for charging activities. Each EV driver is trying to minimize his/her charging cost while making decisions and competing with each other for using the charging stations. This work proposes an optimal charging station placement model which aims at minimizing the congestion in charging stations suffered by all EV drivers. We take into account the bounded rational charging behavior of EV drivers. Through analyzing EV drivers' decision-making in the charging process, we propose a 2-Level Nested LQRE charging behavior model that combines LQRE model and level-k thinking model. We design a set of user studies to simulate the charging scenarios, collect data from human players and learn parameters of the 2-Level Nested LQRE charging behavior model. Experimental results show that our charging behavior model well captures the bounded rationality of human players in the charging process. The selection distribution of all players tends to converge after a number of repeated playing. Furthermore, we formulate the charging station placement problem with the 2-Level Nested LQRE model and design a heuristic searching algorithm to solve it. Our approach obtains placement with a significantly better performance by decreasing more than 8% for the social cost compared with benchmark approaches.

In Chapter 7, we discuss the three works presented in Chapter 4 to Chapter 6 and conclude these works. We also suggest several research topics for future.

Chapter 2

Related Work and Preliminaries

2.1 Charging Station Placement

In the past years, the raising concern of the shortage of non-renewable energy has made new energy a hot research topic. In the transportation domain, the electric vehicle is regarded as an ideal substitute for traditional vehicles.

Many researchers have made efforts in related techniques to enable/speed up the EV diffusion, for example analyzing the key factors that may infect the construction of EV infrastructure [14]. Meanwhile, many researchers are working on integrating EVs into the traditional transportation network, for example with a system for EV integration with energy grid [15]. Rigas et al. gave a survey of such research works [16].

While charging is a premium issue for EV diffusion, placement of charging station-s and charging mechanism are two important topics worth studying. There are some works studying the charging mechanism/pattern based on settled charging network. Rei et al. presented a charging control mechanism for EVs to integrate with the power grid [17]. Bashash and Fathy designed a cost-optimal charging pattern for EVs that want to minimize the cost when charging in a time-varying pricing network [18]. Alesiani et al. focused on the routing problem of EVs when they want to decide the charging destination with consideration of the charging cost, remaining energy etc [19]. In addition, some works focus on new ideas. For example, providing mobile charging rather than charging at changeless places for EVs [20] or designing sustainable transportation rather than merging into the current one [21].

There is also some research on the placement of charging stations. Tan and Lin proposed to site the charging stations mainly concerning the demand flow and its uncertainty [22]. Unfortunately, their work fails to consider the interactive and implicitly competing EV drivers. Timpner and Wolf designed a scheduling strategy in the case EVs are charged in carparks [23]. However, this is not applicable to the general case for the potentially large number of users in the city, because equipping each carpark with charging infrastructure is not realistic. Hausler et al.'s work also combines charging and parking [24]. Baouche et al. modeled the charging stations with a modified Fixed Charge Location Model mixed with a p-dispersion constraint, which is used to minimize the charging cost and construction cost [25]. Although accurate estimation of travel and energy demand was proposed, the authors ignored the influence of the self-directed EV drivers' behavior.

Facility location decisions are commonly required in modern society, like in the planning and construction of public facilities. Due to the high investment, inconvenience in replanning and reconstruction, and the dynamic and unpredictable future environment, facility location is a class of knotty yet challenging problem and has engaged many excellent researchers in this area. In this section, we discuss existing works according to the problem models and proposed algorithms.

The MiniSum models aim to minimize the total cost of the facility placement process, which usually consists of the construction cost of different facilities and (or) the use-cost of all facility users. The relating factors are usually taken into consideration through the existence of constraints. Such models consider the facility location problems as single-level optimization problems [26–29].

Another class of widely used models is MiniMax regret model, which aims to minimize the worst-case (i.e., the maximum) social cost (i.e., the total cost of the investors and facility users) or other cost function (e.g., the weighted distance for users to use their assigned facilities). Existing works that use the Minimax regret model include [30–34].

Bi-level optimization is also frequently used in facility locating since the conflict objectives of investor and facility users can be split into two levels. When the investor decides the facility location decisions, the facility users can make their own decisions in choosing facilities rather than be assigned to corresponding facilities. There have been some works research and use this method to solve the facility location problems [35–39].

Different facility location models satisfy different real-world scenarios. However, more existing work usually attaches no sufficient importance to the target of facility placement: the facility users. They failed in capture the interactions between facility users and the placed facilities. In our research on age-friendly city planning and construction, we will employ bi-level optimization models to capture the problem nature. More specifically, we will use multi-objective bi-level optimization models (more details in Section 2.4.2). However, there is a breach of research on this topic. Therefore, we will analyze the facility location problems in depth and combine knowledge from other areas to make our model more comprehensive (e.g., psychology knowledge for human behavior modeling and transportation research for traffic modeling).

2.2 Facility Management

While there have been some existing work concerning the management of EV charging stations, they mostly focused on the spatial placement of charging stations. For example, Frad et al. studied the placement and capacity allocation of EV charging stations for an area of Lisbon with the emphasis on maximizing coverage of charging demands [40]. Wong et al. proposed a multi-objective planning model for the placement of EV charging stations in Chengdu, China, with a solution based on demand and usage of existing gas stations [41]. Chen et al. particularly considered EV users' costs for accessing charging stations, and minimizing the costs and penalizing unmet demand [42]. Moreover, He et al. and Xiong et al. took a more broad view and emphasized the impact on the overall efficiency of transportation system when optimizing the placement [7, 10]. However, a major drawback of the existing work is that such once-for-all solutions can hardly adapt to rapidly changing urban structures. Development of local infrastructures, such as opening-up of a new hospital, shopping mall, school, or housing estate, can all fundamentally modify the residential traffic pattern, making it unbalanced against the existing charging network. Thus follow-up adjustments are expected but might be costly and inefficient if we only rely on optimizing the placement.

2.2.1 Dynamic Pricing

There have been some works leveraging dynamic pricing to improve the efficiency of public transportation systems, such as taxi systems [3, 4]. Some works have particularly focused on real-time pricing and charge-discharge policy for EV management [5, 6]. However, their aim is just to balance electricity load in power grids, while traffic condition is not in their consideration. Moreover, their method cannot be incorporated with trivial modifications because the traffic condition deeply relates to EV users' self-interested charing behavior associated with a graph-based road network, which is all absent from existing work.

2.3 Congestion Games and Equilibrium

Congestion games have been proved to be potential game. The complexity of congestion game is greatly relevant to the latency function of congestible resources in the congestion game. Existing works mainly study the computation of pure Nash equilibrium of congestion game, the price of anarchy and the price of stability. However, there is a shortage of research on mixed Nash equilibrium. Next, we will review the existing works on congestion game regarding different topics (computation of pure Nash equilibrium, the price of anarchy and price and stability). At the end of this section, we will briefly discuss the mixed Nash equilibrium of congestion game.

2.3.1 Computation of Pure Nash Equilibrium

Congestion games defined by Rosenthal in 1973 [43] are a class of games with players and resources, while the resources are congestible, i.e., the cost of using a resource depends on the number of users that use it. Rosenthal also proved that any congestion game is a potential game, while the converse that a potential game always has a congestion game with the same potential function [44]. With the help of potential functions, the existence of pure Nash equilibrium in congestion games can be proved. The computation of pure Nash equilibrium has been studied by researchers since congestion game was defined [45–51].

2.3.2 Price of Anarchy and Price of Stability

Price of anarchy and price of stability are important for measuring the performance the Nash equilibrium comparing with the idealized social optimal solution which has no consideration of the players' strategic decisions. Price of anarchy is the ratio of the worst-case social cost among the Nash equilibria to the idealized social optimal, while the price of stability is the ratio of the best-case social cost among the Nash equilibria to the idealized social optimal solution. Studies on price of anarchy of pure Nash equilibria are presented in some works [52–59]. These works concentrate on the linear or polynomial congestion games, where the price of anarchy can be proved an upper bound 2.618. Price of stability is studied in [54, 57, 59–61].

2.3.3 Mixed Nash Equilibrium

As far as we know, there is no existing work studying the mixed Nash equilibrium for congestion games. One reason is that in most scenarios, it is reasonable to assume that the players consult pure equilibrium strategies, i.e., one player uses one fixed strategy. However, this can be unrealistic in some real-world problems. For example, in the electric vehicle charging station placement problem, the electric vehicle drivers do not have to stick to the same charging station, on the contrary, they will choose different charging stations according to various elements. A class of congestion game, named non-atomic game [48, 59], consider this case but still use pure Nash equilibrium to compute the strategy distribution rather than mixed Nash equilibrium.

Existing researches mostly study on the linear/quadratic congestion games and stick to pure Nash equilibrium. We assert that this is not realistic in some real-world scenarios (e.g., the electric vehicle charging station placement problem). Our research will study the congestion game with non-quadratic or even non-convex congestion function according to the reality. At the same time, we will work on the mixed Nash equilibrium considering the user features of some kind of facilities.

2.4 Bi-Level Optimization

Research on the challenging bi-level Optimization first appeared in 1973 and named as Mathematical Programs with Optimization Problems in the Constraints by Bracken and McGill [62]. Since then, there has been a bunch of work on applications and algorithms of bi-level optimization emerged. Generally speaking, bi-level optimizations are often transferred to single-level optimizations for computing solutions by using exact or approximate methodologies to replace the sub-level optimization with constraints. We review the existing works on such bi-level optimization methods in the first part, while the second part will be the discussion on the existing work on studies of a more complex class of bi-level optimization: multi-objective optimization.

2.4.1 Transformation to Single-Level Optimization

To solve the bi-level optimization through problem transformation from bi-level to single-level, in classic exact approaches, researchers usually make the assumption on s-moothness, linearity, and convexity. There are three classes of classic approaches, KKT conditions, branch-and-bound and penalty methods. Meanwhile, a class of evolutionary approaches is being studied and applied to compute optimal solution for bi-level optimization. We review the existing work according to the approaches employed as follows.

Karush-Kuhn-Tucker (KKT) optimality conditions are frequently used in bi-level optimization. Bianco et al. use a linear bi-level optimization to formulate a transportation network design problem and solve it by using KKT conditions and complementary constraints linearization, which helps to transform the bi-level optimization into single-level optimization [63]. Similar approaches that use KKT conditions to eliminate the sub-level optimization and transform the bi-level optimization into single-level optimization are used for linear [64, 65], convex [66–69] and even non-convex [70] bi-level optimization problems.

Branch-and-bound (B&B) approach is another class of approach for solving bi-level optimization in a classical way. B&B is an algorithm designed for discrete and combinatorial optimization problems first proposed by Land and Doig [71] for non-convex discrete programming. The basic idea of B&B is to split the feasible region into subregions and compare the optimal solutions (e.g., minimum objective) in different subregions. Once there is a subregion A, where the lower bound of the objective for variables in A is greater than the upper bound of the objective for variables in another subregion B, the subregion A can be safely discarded in the search. B&B approach is applied for solving linear [64, 65], convex [66, 72–74] and nonconvex [71] bi-level optimization problems. Some of these works combine B&B with KKT conditions to solve bi-level optimizations [64–66]

Another class of classic approach for transforming bi-level optimization into single-level optimization is called penalty methods. The initial step of penalty method in bi-level optimization was achieved by Aiyoshi and Shimizu [75, 76] and Shimizu and Aiyoshi [77]. The penalty methods replace the sub-level optimization problem by the penalized problem, which adds a large value for unsatisfied constraint and otherwise 0 to the minimizing objective. Penalty methods are used for linear [78–81], convex [82, 83] and noconvex [84] bi-level optimizations. Some works combine the penalty methods with KKT conditions in solving bi-level optimization problems [78, 81].

Evolutionary techniques have been developed in recent years. The general format of solving bi-level optimization with evolutionary techniques is to solve the upper-level optimization and sub-level optimization iteratively. One advantage of using evolutionary techniques is that these methods do not adhere to the simplifying assumptions required by classic exact methods. Such methods have been used to solve non-convex optimization problems in [85–88] etc.

2.4.2 Multi-Objective Bi-Level Optimization

Despite the extensive number of complex real-world problems that can be formulated as general bi-level optimization problems, there are some problems even more complicated such that a comprehensive model is necessary to be multi-objective, which means that the problem can have multiple objectives at one or both levels. There have been several research works on the multi-objective bi-level optimization problems. For example, Echfelder expresses the feasible points for the upper-level problem as a solution set of a multi-objective optimization problem and then approximates the solution set using results from the multi-objective optimization [89]. Evolutionary optimization is also employed in relating works [90–94].

Bi-level optimization problems (both the general format ones and the generating multi-objective ones) are important research topics in both mathematical programming and operation research communities. As far as we can see, although there have been some dedicated algorithms for some classes of bi-level optimization problems, efficient and comprehensive researches on other classes are still in need. There are three challenges still remain intractable: 1) for non-convex bi-level optimization problems, which are common in real-world scenarios, we still need efficient exact approaches; 2) multi-objective bi-level optimization problems add an additional stratum of difficulty for researchers; 3) there are no mathematical tools for bi-level optimization problems like CPLEX and KNITRO for single-level optimization problems. In our proposed research, it is inevitable for us to encounter multi-objective bi-level optimization problems. We aim to make contributions to this area, especially in the above three aspects.

2.5 Bounded Rational Human Behavior Study

When EV drivers make decisions for charging EVs, they are self-interested and competing with each other to use the chargers in charging stations. Congestion game is a natural method to model this scenario with EVs as players and charging stations as congestible resources. EVs' charging behavior (i.e., the decision on using which charging station) influences the performance of charging stations, which in turn impacts their decisions.

Perfect rationality has been extensively studied and used to model players' decision-making in congestion games [95, 96]. Nash equilibrium (NE) in a game is defined as the state where no players can improve his/her utility by unilaterally changing his/her own decision. While the number of players goes to infinity, NE converges to the Wardrop user equilibrium (UE), i.e., whichever choice used by the players has the same and maximum utility. However, the assumptions in perfect rationality are usually impractical in reality due to players' lack of accurate information (on others' behavior) and computational ability. Bounded rationality is first proposed by Simon [97], where players tend to seek a satisfactory solution rather than an optimal one. However, the qualitative definition of "satisfactory solution" does not specify its distance from the optimal solution and thus it is hard to quantitatively evaluate it for specific problems. Moreover, the existence of bounded rational user equilibria (BRUE) makes the solution space a non-convex set.

The CSPP has been widely studied in past years. Most of them[8, 9, 40–42, 98–101] ignore the interdependent influence of the participating EV drivers' behavior on the performance of the charging station placement. Among the few works that consider EV drivers' behavior, He et al. [7] use a multinomial logit model to model the EV drivers' charging route distribution. However, they fail to explain why the drivers' behavior would form the distribution and how to select the parameter in the logit model. Xiong et al. [10] assume that the drivers are fully rational in the charging game and would form Nash equilibrium in choosing charging stations, which is usually impractical in real-world scenarios.

In this work, we aim to study the bounded rationality of EV drivers in their charging activity and integrate our obtained realistic charging behavior model into the CSPP. We build a charging behavior model for the EV drivers based on LQRE model and level-k thinking model. We learn the EV drivers' preference over different choices from human data and reveal the bounded rationality of players.

However, the assumptions in perfect rationality are usually impractical in reality due to players' lack of accurate information (on others' behavior) and computational ability. Bounded rationality is first proposed by Simon [97], where players tend to seek a satisfactory solution rather than an optimal one. However, the qualitative definition of "satisfactory solution" does not specify its distance from the optimal solution and thus it is hard to quantitatively evaluate it for specific problems. Moreover, the existence of bounded rational user equilibria (BRUE) makes the solution space a non-convex set.

2.5.1 Quantal Response Equilibrium

To model the bounded rationality of human players, Mckelvey and Palfrey [102] propose Quantal response equilibrium (QRE). QRE specifies a set of mixed strategies for each player while assuming a random perception error in utility estimation. A typical QRE formation is the logit equilibrium (LQRE) (refer to Eq.(2.3)) based on an presumed error distribution, i.e., i.i.d. Gumbel distribution. Note that the subscript i denotes a specific choice, u_i is the utility of choice i, and p_i is the probability of using choice i.

$$p_i = \frac{e^{\lambda u_i}}{\sum_j e^{\lambda u_j}} \tag{2.1}$$

However, the rationality parameter $\lambda \in (0, \infty)$ defined in QRE may vary from problem to problem, and from experiment to experiment. This character hinders the application of QRE in real-world problems. Some works (e.g. [103]) discover that value of λ is largely dependent on specific problem structure, but there is no further research on how it is influenced.

Yang et al. [104] used QRE to model the bounded rationality of adversaries. Pita et al. [105] used a robust optimization technique named MATCH to compute the optimal strategy of defenders. Nguyen et al. [106] later integrated a subjective utility function with the QRE model.

However, the existing works using QRE to study human behavior treat the rationality parameter as a constant. While we apply this rule in the experimental analysis of our designed simulations, we find that it fails to fit the real data. Intuitively, the

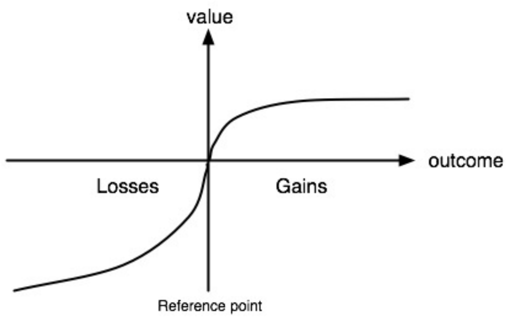


FIGURE 2.1: The value function of prospect theory

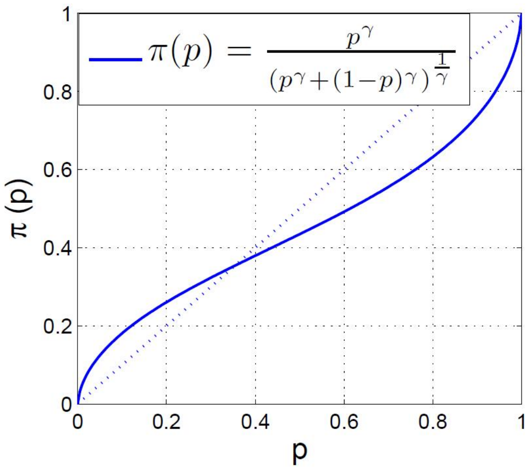


FIGURE 2.2: The weighting function of prospect theory

rationality of a human being should be highly dependent on the problem complexity, i.e., people can make a rational choice when faced with a very easy situation, but it becomes harder when the situation becomes complicated. Eliaz [107] studied the influence of complexity (which indicates the response rule) on players' decision making in games; Diasakos [108] studied the relationship between the bounded rationality and the searching cost of alternatives, which was estimated by the number of machine states. However, none of them modeled the rationality level of human players as a function of the problem complexity.

2.5.2 Prospect Theory and Cognitive Hierarchy Theory

Human behavior has also been valued and extensively studied in the economic research community. Prospect theory, a Nobel-prize-winning theory is a classic behavior economic theory proposed by Daniel Kahneman and Amos Tversky [109]. Prospect theory (as shown in Figures 2.1 and 2.2) describes that people's prospect utility for a certain decision is influenced by a reference point, as well as the loss or gain versus it. However, competition among people is not in the consideration of prospect theory. Another behavior model, cognitive hierarchy model (which is well known for Level-k thinking model [110]) assumes that players act with different levels of rationality.

2.6 Preliminaries

In this section, we present some concepts and theorems from Game theory. Our approach is mostly based on these concepts.

Game theory is the study of strategic decision making, which is widely used in economics, political science, psychology, biology, logics and computer science. Three elements of a game are players, strategies, and payoff. Figure 2.3 shows a typical two-player game, where there are two players and the players respectively have m and n strategies. Different outcomes in the cells give different payoffs for each player, where the number in the left is the payoff of player 1 and the right is for player 2. A game of real-world is usually much more complex than the example. For example, the payoff for each player can be a complex function of time and both players' strategies. In some cases, players in a game choose one pure strategy (i.e., each player consults to one strategy), but in other cases, players can consult to a mixed strategy, i.e., play different strategies with a probability distribution. Nash equilibrium is the most basic solution concept for such kind of non-cooperative games, where two or more players choose their best strategies without consideration of cooperation reciprocality. Each player is assumed to know the equilibrium strategies of all other players and has nothing else to gain by changing only its own strategy in Nash equilibrium.

Congestion games are a class of games proposed by Rosenthal in 1973 [43]. The motivation of congestion games comes from traffic scenarios, where the payoff of players depends on the load of the resources that they select to use. The definition of a discrete congestion game is described below.

	Player 2 Strategy 1	...	Player 2 Strategy n
Player 1 Strategy 1	1, 3	...	4, 2
⋮	⋮	⋱	⋮
Player 1 Strategy m	0, -1	...	-1, 4

FIGURE 2.3: Payoff matrix of a normal game with two players

Definition 2.1. (Congestion Game) A discrete congestion game consists of following components:

- A basic set of congestible elements E;
- n players;
- A finite set of strategies S_i for each player i, where each pure strategy $P \in S_i$ is a subset of congestible elements E;
- The load x_e of a congestible element e is decided by all players strategies, i.e., $x_e = \#\{i : e \in P_i\}$;
- The payoff $v_e(x_e)$ of element e is dependent on its load x_e;
- With strategy P_i, player i gets payoff $\sum_{e \in P_i} v_e$.

Since congestion games are a special case of potential games, there are always Nash equilibria (defined below for above-defined congestion game) for congestion games.

Definition 2.2. (Nash Equilibrium) In the congestion game, let P be the strategy profile for all n players, where P_i is the strategy for player i and P_{-i} is the strategy for all players except player i. Under each strategy profile, each player i has payoff $V_i(P) = \sum_{e \in P_i} v_e$. A strategy profile P^* is a Nash equilibrium if no player can increase his/her payoff by unilaterally changing his/her strategy, i.e.,

$$V_i(P_i^*, P_{-i}^*) > V_i(P_i, P_{-i}^*), \forall i, \forall P_i \in S_i. \tag{2.2}$$

When the number of players $n \to \infty$, the discrete congestion game can be treated as a continuous congestion game (also named non-atomic congestion game). In this case, one player is considered to be "infinitely small". The equilibrium state P^* of continuous congestion game can be redefined as $v_e^* = \min_{e \in E} v_e, \forall e^* \in P^*$.

The traditional game theory assumes full rationality from players, which means human players can compactly know the strategies of all players and compute the optimal strategy accordingly. However, this is nearly impossible due to a number of limitations. In reality, players are always bounded rational. To model the bounded rationality of human players, Mckelvey and Palfrey [102] propose Quantal response equilibrium (QRE). QRE specifies a set of mixed strategies for each player while assuming a random perception error in utility estimation. A typical QRE formation is the logit equilibrium (LQRE) defined below.

Definition 2.3. (Logit Quantal Response Equilibrium) In logit quantal response equilibrium, players form a strategy distribution based on a presumed error distribution, i.e., i.i.d. Gumbel distribution. The probability of players selecting strategy i is p_i as stated with Equation (2.3), where u_i is the payoff of strategy i and λ is called a rationality parameter for the players.

$$p_i = \frac{e^{\lambda u_i}}{\sum_j e^{\lambda u_j}} \tag{2.3}$$

When $\lambda \to 0$, players tend to be irrational and choose one option randomly based on a uniform distribution; when λ increase, players tend to be more rational; $\lambda \to \infty$ means that players are full rational and would all use the optimal strategy (in this case, LQRE converge to Nash equilibrium).

Bi-level optimization is a class of optimization problems, where an optimization is embedded in another (be part of the constraints). The outer optimization task is usually named upper-level optimization task, while the inner one is referred to as sub-level optimization. Variables exist in both levels. A general formulation of bi-level optimization can be represented as follows.

$$\min_{\mathbf{x}\in X,\mathbf{y}\in Y} \quad F(\mathbf{x},\mathbf{y}), \tag{2.4}$$

$$\text{s.t.} \quad G_i(\mathbf{x},\mathbf{y}) \leq 0, \forall i \in \{1,...,I\}, \tag{2.5}$$

$$H_k(\mathbf{x},\mathbf{y}) = 0, \forall k \in \{1,...,K\}, \tag{2.6}$$

$$\mathbf{y} \in \arg\max_{\mathbf{z}\in Y} f(\mathbf{x},\mathbf{z}), \tag{2.7}$$

$$\text{s.t.} \quad g_j(\mathbf{x},\mathbf{z}) \leq 0, \forall j \in \{1,...,J\}, \tag{2.8}$$

$$h_l(\mathbf{x},\mathbf{z}) = 0, \forall l \in \{1,...,L\}. \tag{2.9}$$

In above formulation, there are two classes of variables, $\mathbf{x}$ and $\mathbf{y}$ respectively refer to the upper-level problem and sub-level problem. F and f respectively represent the optimization objective for upper-level optimization and sub-level optimization, meanwhile a series of inequality and equality constraint functions for upper-level (and sub-level) optimization are indicated by G (and g) and H (and h), respectively.

In practice, bi-level optimization commonly occurs in a number of real-world problems, including domains of transportation, economics, decision science, business, engineering and environmental economics. In our research, the facility location problem has two sides, i.e., the investor and the facility users, thus we will formulate the facility location problems as bi-level optimization problems with the objective of investors (e.g., the government) as the upper-level optimization task and the objective of facility users as the sub-level optimization task.

Chapter 3

Optimal Electric Vehicle Fast Charging Station Placement

Fossil fuels are generally considered as non-renewable resources and the running out is only a matter of time. Meanwhile, the environmental problem caused by burning the fossil fuels is aggregating. Therefore, it has been an arisen topic to study and use alternative energies. Transportation is the main consumer of fossil fuel energy and contributes a large proportion to the pollution. Electric Vehicles (EVs) are promising to replace traditional internal combustion vehicles and move pollution away from urban areas. Electricity is efficiently transformed from both fossil fuels and renewable energies (e.g., solar energy and tidal energy). Thus EVs on the road can achieve zero emission and reduce the pollution from transportation. In recent years, there has been a rapid growth of studies on EVs accompanying with the rising popularity of the smart city concept [111]. A top-priority element for efficient and fast diffusion of EVs is the support of charging facilities like fast charging stations. Although charging at home is an alternative for the EV users, it costs too much time (which can reach 6 to 8 hours). Charging stations with high voltage [112] is then a necessity for the convenience of EV users because it can charge the EVs at least 12 times faster. The EV drivers' convenience of charging is highly dependent on the distribution of charging stations. Thus the latter can affect the public's willingness to choosing EVs, and the EV drivers' charging behaviors. Consequently, the traffic conditions in the road network and the charging system's performance are also influenced.

To study the Charging Station Placement Problem (CSPP) realistically, we consider the self-interested charging behaviors of EV users, which are competitive and strategic. The interaction of charging behaviors with environmental factors including traffic condition in the road network and queuing condition in charging stations are also formulated into the model to decide the optimal charging station placement. There are mainly three reasons for such consideration. First, the queuing condition in charging stations is considered because the queuing experience in charging stations is proven to be significant for the adoption of EVs [1, 2]. Second, inspired by the works of Gan et al. [3, 113], we can see that the traffic congestion is influential in car drivers' driving activity, especially during peak hours. Thus we plan the charging stations based on the peak hour traffic network to minimize the charging activities' influence on the traffic condition. Third, since the EV users' cannot be centralized, we need to analyze how their charging behaviors are influenced by factors like distribution and size of charging stations and traffic condition.

This chapter makes three main contributions. Firstly, we build a realistic CSPP model, in which the EV drivers' strategic charging behaviors, the traffic condition and the queuing time in charging stations are considered. The overall objective is set as minimizing the total charging cost of EV drivers (named social cost), and EV drivers are assumed to minimize their charging cost with strategic charging behavior. We formulate the CSPP as a bi-level optimization problem, where we take the social cost as the upper-level objective, which is the goal of the government (who is assumed to be the one to decide the placement of charging stations); a charging game (which falls into the class of congestion games) is formulated in the sub-level problem and Nash Equilibrium is adopted to define the EV drivers' charging behaviors. Secondly, we successfully transfer the bi-level optimization problem into an equivalent single-level optimization problem by analyzing the definition and structure of the Nash Equilibrium in the charging game. We propose the algorithm OCEAN (Optimizing eleCtric vEhicle chArging statioN placement) to compute the optimal charging station placement. However, the real-world problems have a large scale of variables, and OCEAN is unable to handle them due to the existence of integer variables and the huge variable space. Thus we furthermore work out a heuristic algorithm OCEAN-C (OCEAN with Continuous variables) that can handle the real-world CSPP and ensure solution quality. Thirdly, we design and execute a lot of experiments for both mock data and the real situation of Singapore. The experimental results prove that the designed algorithms OCEAN-C can efficiently solve the CSPP and our approach outperforms some typical baseline methods.

3.1 Charging Station Placement in Singapore

Named as "Garden City", Singapore has a good reputation for its nice environment and air quality. However, there is still pollution around here, especially the air pollution caused by the heavy traffic that surrounds us all day. According to the official data [11], 20% of the total carbon emission and 75% of the air pollution in Singapore are caused by the land transportation system, mostly attributed by the motorized traffic. As a result, Singapore government is working on mitigating the environment problem due to the traffic system by introducing the clean EVs as a replacement for traditional internal combustion vehicles. The Singapore authorities have started to test the possibility and feasibility of introducing EVs into Singapore since 2011. As a metropolis with advanced energy network, the electric-car manufacturer BYD Asia-Pacific announced that Singapore has the "best potential" to implement EVs [12].

The construction of EV charging stations is the first challenge, to which the government needs to rise for the successful introduction of EVs. Besides the finance concern, there are some elements far more important and urgent, among which traffic condition is of top priority. The planning of charging stations calls for a careful investigation to avoid aggravating the traffic congestion of this small city. Specifically, Singapore is a small metropolis with a very small territory of $718.3km^2$ only (Figure 1.1). The maximum east-west distance is $42km$, while the north-south distance is barely $23km$. In a city like Singapore, the most commonly considered problem, namely the limited EVs mileage (usually above $100km$ and some can exceed $500km$ [13]) is not a big issue. Contrasting with the small territory of Singapore is its huge population, which also means a large population of vehicles. According to the Singapore official announcement, there are more than $970,000$ motor vehicles in the year 2014 on this small island, which indicates the heavy traffic. The fact implies that rather than limited mileage, we should make more efforts on balancing the traffic in consideration of EV users' charging behaviors.

Our first consideration is to minimize the traffic congestion caused by the charging activities in the process of planning the distribution of charging stations. The traffic condition is influenced by charging activities of all the EV drivers. In return, it also influences the EV drivers' decision making of choosing charging destinations. Moreover, the queuing time in charging stations is also studied as a vital element that affects EV drivers' charging decisions. One reason is the long queuing time implies larger space required to accommodate queuing EVs in charging stations. Another is that it would frustrate the EV drivers. We model the interactions among the allocation of charging stations, EV drivers' strategic and self-interested charging activities, traffic congestion on the roads and queuing time in charging stations to formulate the CSPP realistically. To compute the optimal solution, we propose OCEAN and an efficient heuristic algorithm OCEAN-C.

3.2 Charge Station Placement Problem

To minimize the social cost (defined in Section 3.2.3), we try to find the optimal charging station placement in a region. In the following, we first define the topology of the studied region and then explain how we define the charging cost of the EV users. A congestion-game-based interpretation of the CSPP is introduced afterwards, which is followed by a bi-level optimization formulation. For better understand of the definitions, we present all the notations used in problem definition section in Table 3.2.

3.2.1 Zones and Charging Stations

We divide the region to be analyzed into n zones in set $\mathcal{N} = \{1, 2, ..., n\}$ according to the geographic and residential condition. We assume each zone is a candidate for

TABLE 3.1: Notation overview

Notation	Description
$\mathcal{N} = \{1, ..., n\}$	The set of n zones (i.e., the charging station candidates)
γ_i	The number of resident EV drivers in zone i
x_i	The allocated number of chargers in zone i
B	The budget of chargers to be allocated
$A = \{a_{ij}\}$	The adjacent relationship among zones
$R = \{\langle i, j \rangle\}$	The set of roads with $a_{ij} = 1$
$D = \{d_{ij}\}$	The distance between pairs of zones
α_{ij}^0	The background congestion on road $\langle i, j \rangle$
α_{ij}	The congestion on road from zone i to zone j with consideration of the charging EVs
y_{ij}	The number of EVs from zone i, charge in j
$y_j = \sum y_{ij}$	The total number of EVs charge in zone j
f_{ij}	The travel time cost of an EV $\in y_{ij}$
λ	Parameter in travel cost function
$1/\tau$	The proportion of EVs charge during peak hours
k_{ij}	The inverse of road capacity (used for α_{ij})
μ	The serve capacity per charger per unit time
g_i	The queuing time in charging station i
$\mathbf{p}_i = \{p_{ij}\}$	The charging strategy of EVs in zone i
$\mathbf{P}(\mathbf{P}_{-i})$	The strategy profile of all EVs (except EVs from zone i)
C_i	The charging cost of all EVs in zone i

building charging station. The specific position of the station can be decided through preliminary studies, which is out of our consideration. For simplification, we name the candidate position as the center of the zone. In the following, we also use the set of zones to represent the set of changing station candidates. Any pair of zones are treated as adjacent if they share a geographical border and they are directly connected by the main road. The matrix $A = \{a_{ij}\}_{n \times n}$ is used to represent the adjacency relationship between different zones, where $a_{ij} = 1$ and $a_{ij} = 0$ respectively represents that zone i and zone j are adjacent or nonadjacent. For the ease of notations, we define a zone to be adjacent to itself, i.e., $a_{ii} = 1$. The matrix $D = \{d_{ij}\}_{n \times n}$ denotes the distances between pairs of zones. The average length of trips of EV drivers that reside in zone i and charge in zone j is d_{ij}, which is estimated by the distance between their centers, and d_{ii} is set as the radius of zone i. The concrete example of this thesis is Singapore. According to the conventional partitioning method from the official site, we divide it into a number of zones as shown in Figure 1.1.

3.2.2 The EV Model

Although EVs can be charged at home, some EV drivers would still need charging stations because (1) not everyone has his/her own garage to charge the EV and (2) they might forget to charge during the night (since charging at home is time-consuming) and need fast charging. We assume the number of resident EV owners in need of charging in charging stations in zone i as γ_i. The size of the charging station to be built in zone i, i.e., the number of chargers is denoted as x_i, which is to be decided in this work. Note that x_i is an integer and can be 0 (meaning that no charging station is built here). Intuitively, EV owners are not willing to drive too far to charge. To verify whether the assumption is believable, we relax it and allow EVs to charge in nonadjacent zones in experiments as described in Section 3.4.2.4. The results prove it to be acceptable. Thus we assume that EV drivers can choose any one from adjacent zones to charge. The number of EVs that charge in the zone i during peak hours is denoted as y_i. Assume that electricity prices are the same for different charging stations, different charging destinations are indifferent to financial cost. Thus we only consider the time cost for EV drivers, including the travel time and the queuing time.

Travel time. We consider the distance d and traffic condition α (i.e., congestion level) on the road as two factors that influence the travel time. The relationship between travel time f and the two factors is shown with Equation (3.1), where λ is a constant [114].

$$f_{ij} = \lambda d_{ij} \alpha_{ij} \tag{3.1}$$

The congestion level α depends on the traffic on the road and is defined in Equaion (3.2) following transportation science research [115–118]. When there is more than one road directly leading from zone i to zone j, we use the average traffic condition, road capacity, and distance. We use α_{ij}^0 to denote the background traffic congestion, i.e., the normal traffic congestion caused by any other vehicles except the EVs heading for charging.

$$\alpha_{ij} = \alpha_{ij}^0 + k_{ij} y_{ij} / \tau \tag{3.2}$$

Note that k_{ij} denotes the inverse proportion of the road capacity; the charging flow from zone i to zone j is represented by y_{ij}; and the fraction of EVs that charge during peak hours is set as $\frac{1}{\tau}$. Thus $k_{ij}\frac{y_{ij}}{\tau}$ represents the congestion caused by EV users' charging activities. The congestion level within zone i is α_{ii}, set as the average congestion level of the main roads inside zone i. We focus on the traffic condition and charging demand during peak hours because (1) the traffic congestion is usually most serious during peak hours period and (2) there are some EV users have to charge during this period due to their limited time and urgent energy demand.

Queuing time. Besides the traffic condition, we also consider EV users' charging activities' influence on the queuing time in charging stations during peak hours. Recalling that we assume that 1 in every τ EVs would charge at charging stations during peak hours, we use $\frac{y_i}{\tau}$ to denote the number of EVs that arrive in zone i for charging during peak hours. We assume that the average queuing time of EV users is directly proportional to the number of EVs in the same station, which can be formally defined as

$$g_i = y_i/\mu\tau x_i. \tag{3.3}$$

Note that we use μ to denote the serving rate of chargers, i.e., the number of EVs can be served per charger per unit time.

3.2.3 A Congestion-Game-Based Interpretation

As we can see from the definition of charging cost in Section 3.2.2, when the background traffic (i.e., the corresponding parameters) and the charging station placement (i.e., the number of chargers in each zone) are fixed, the travel time and queuing time both are decided only by the number of EVs that are using this corresponding road or charging station. We can treat the roads and charging stations as congestible resources. Thus EV users are playing a charging congestion game [119]. We formally define the components of the charging game in the following.

- **Congestible element**. There are two sets of congestible elements in the charging game, respectively the charging stations (i.e., the set of zones), which are denoted as $\mathcal{N} = \{1, ..., n\}$ and the roads (among pairs of adjacent zones and inside each zone) denoted by $\mathcal{R} = \{\langle i, j\rangle | i, j \in \mathcal{N}, a_{ij} = 1\}$. Note that a road $\langle i, j\rangle$ is sensitive to the direction and represents the road leading from zone i to adjacent zone j.

- **Player.** We regard the γ_i EV users in the same zone as identical players with the same strategies.

- **Strategy.** For each player i, we assume that a pure strategy is to charge in a zone j adjacent to zone i (this can also be assumed as a set of en-route zones), i.e., to use congestible elements charging station i and corresponding road $\langle i, j\rangle$. The players can play mixed strategies, which means the group of EVs in the same zone charge with different pure strategies and their choices form a distribution. Formally, the probability that EVs in zone i charge in zone j is denoted as p_{ij} and the mixed strategy of player i is defined as $\mathbf{p}_i = \{p_{ij}\}$. For example, a group of EV drivers in zone i can charge in 4 different zones as shown in Figure 3.1 Then the strategy profile of all players are denoted as $\mathbf{P} = \langle \mathbf{p}_i \rangle$.

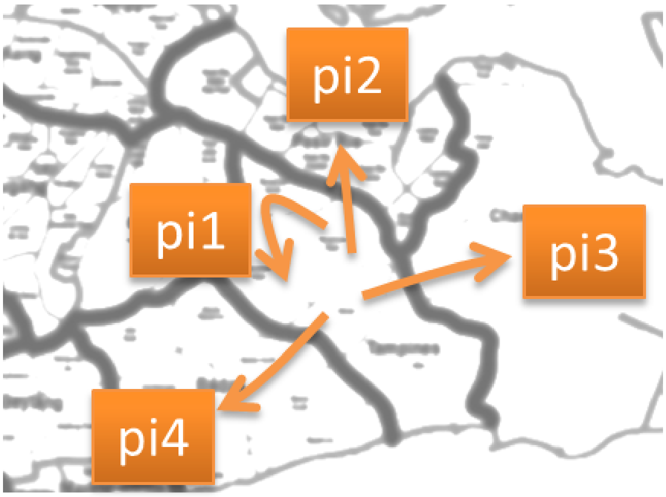


FIGURE 3.1: Strategy demonstration

- **Cost**. The congestion cost for each congestible element is defined in Eqs. (3.1) and (3.3) respectively for $i \in \mathcal{N}$ and $\langle i, j\rangle \in \mathcal{R}$. For simplicity, we use $g_i(\cdot)$ and $f_{ij}(\cdot)$ to denote the congestion cost, whose variable is the number of users for corresponding congestible element. According to the players' strategy profile, we can then derive the number of users of each congestible element. For congestible elements $\mathcal{R}$ and $\mathcal{N}$, the number of users under strategy profile $\mathbf{P}$ is respectively:

$$y_{ij} = \gamma_i p_{ij}, \tag{3.4}$$

$$y_j = \sum_{i \in \mathcal{N}} y_{ij}. \tag{3.5}$$

Next, we can define the charging cost of each player i according to the derived cost for each congestible element. For easy notation, we denote the set of adjacent zones of zone i as $\mathcal{A}_i = \{j | a_{ij} = 1\}$. Then the charging cost of player i, i.e., C_i can be formulated as a function of the strategy profile $\mathbf{P}$ as in the following

$$C_i(\mathbf{P}) = \sum_{j \in \mathcal{A}_i} \gamma_i p_{ij}(g_j(y_j) + f_{ij}(y_{ij})). \tag{3.6}$$

3.2.4 Bi-level Optimization Formulation

For the solution concept of the above charging congestion game, we adopt the mixed strategy Nash equilibrium concept. Specifically, with the assumption that all the players are aware of other players' charging strategies, under the equilibrium state, no player can decrease her charging cost by unilaterally changing her own charging strategy. Formally, we can define the equilibrium state with a set of optimizations

$$\mathbf{p}_i \in \arg\min_{\mathbf{p}'_i} C_i(\mathbf{P}_{-i}, \mathbf{p}'_i), \forall i \in \mathcal{N}.$$

Note that we use $\mathbf{P}_{-i}$ to denote the strategy profile of players except player i (i.e., type i EVs).

When planning the charging station placement, we stand with the government authority, whose goal is to minimize the social cost when given a fixed budget, a number B of chargers. Consider the overall benefits, we define the social cost as the total charging cost of all players (we are able to extend our work to handle other kinds of social cost function, like the financial cost), which can be formally defined as the following formulation when given a charging station placement plan.

$$C(\mathbf{P}) = \sum_{i \in \mathcal{N}} C_i(\mathbf{P}). \tag{3.7}$$

Note that the social cost is a function of the charging strategy of all players, i.e., $\mathbf{P}$ because their strategies influence the charging cost of each of them and sequentially the social cost.

Considering that the government authority wants to decide the best charging station placement $\mathbf{x}$ for the minimal social cost regarding the players' equilibrium in the charging game, we can formulate the CSPP as the following bi-level program **P1**. Equation (3.8) is the objective; Equation (3.9) is for the budget constraint; Equation (3.10) computes the equilibrium strategies of the EV drivers; and the other equations are constraints for the strategies, including the positivity and the 1-sum property. Note that now $C_i(\mathbf{P})$ is also a function of $\mathbf{x}$, but we omitted that in the expression for simplicity.

$$\textbf{P1:} \quad \min_{\mathbf{x},\mathbf{P}} C(\mathbf{P}), \tag{3.8}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{N}} x_i \leq B, x_i \in \mathbb{N}, \tag{3.9}$$

$$\mathbf{p}_i \in \arg\min_{\mathbf{p}_i'} C_i(\mathbf{P}_{-i}, \mathbf{p}_i'), \forall i \in \mathcal{N}, \tag{3.10}$$

$$\sum_{j \in \mathcal{A}_i} p_{ij} = 1, \forall i \in \mathcal{N} \tag{3.11}$$

$$p_{ij} = 0, \forall i \in \mathcal{N}, \forall j \notin \mathcal{A}_i, \tag{3.12}$$

$$p_{ij} \geq 0, \forall i, j \in \mathcal{N}. \tag{3.13}$$

We compute the optimal charging station placement with the Nash equilibrium that can achieve the best social cost. When there are multiple Nash equilibria for a placement, the governor can take steps to lead the EV users to form the best equilibrium with the lowest social cost. For example, the governor can provide bounty for some behaviors. Similar idea is studied widely in security games named tie-breaking [120].

3.3 Solve the CSPP

After we formulate the charging station placement problem as a bi-level optimization problem **P1**, we focus on the algorithm to solve it. The flow of our approach is presented with Figure 3.2. From problem **P1** we can see that the sub-level optimization has multiple objectives, each of which is the objective for a type of EV users in the charging game. This feature makes the problem complicated and unable to be handled with existing solvers. Therefore, we first work on the sub-level optimization problem (Equations (3.10) – (3.13)) and propose an efficient approach to transfer the sub-level optimization problem into a number of constraints, which can restrict the Nash equilibrium space of the charging game (i.e., the solution of the sub-level optimization problem). Then we can result in an equivalent single-level optimization, which is still difficult due to a large number of variables (including integer variables and continuous variables) and large searching space of the integer variables. We propose a searching algorithm for the single-level optimization problem to speed up the computation. Next, we start with analyzing the Nash equilibrium criterion in the formulation, which is useful for problem reformulation.

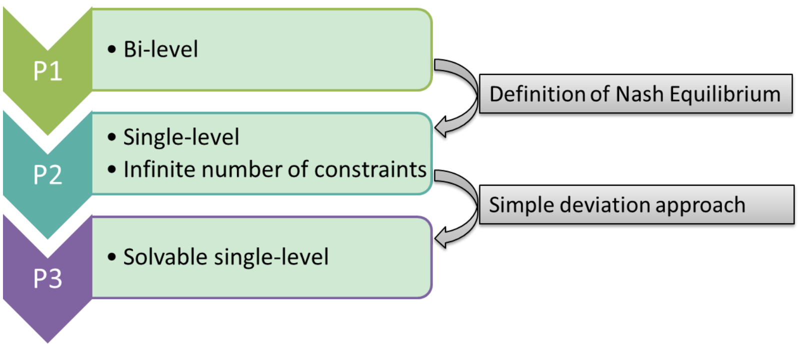


FIGURE 3.2: Approach flow

3.3.1 Deviation of Strategies

According to the definition of equilibrium that we mentioned in above section, we need to consider the player's unilateral strategy change and the influence in its charging cost to prove the stableness of the equilibrium state. Here we use an n-dimensional vector $\Delta\mathbf{p} = (\Delta_1, ..., \Delta_n)$ to denote player i's unilateral strategy change with reference to the strategy profile $\mathbf{P}$. The strategy change is named strategy deviation and should meet the following criterions.

$$\sum_{j \in \mathcal{N}} \Delta_j = 0 \tag{3.14}$$

$$-p_{ij} \leq \Delta_j \leq 1 - p_{ij}, \forall j \in \mathcal{A}_i \tag{3.15}$$

When type i players change their strategy from $\mathbf{p_i}$ to $\mathbf{p_i}' = \mathbf{p_i} + \Delta\mathbf{p}$, recall that y_{ij} denotes the charging flow from zone i to zone j and y_j denotes the number of EVs that charge in zone j, we have $y'_{ij} = y_{ij} + \gamma_i \Delta_j$, $y'_j = y_j + \gamma_i \Delta_j$, and the change in type i EVs' cost can be formulated as:

$$\begin{aligned}
&\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = C_i(\mathbf{P}_{-i}, \mathbf{p_i}') - C_i(\mathbf{P}) \\
&= \sum_{j \in \mathcal{A}_i} \gamma_i [p_{ij}(\lambda d_{ij} k_{ij} \frac{\gamma_i \Delta_j}{\tau} + \frac{\gamma_i \Delta_j}{\mu \tau x_j}) \\
&\quad + \Delta_j(\lambda d_{ij} \alpha_{ij} + \lambda d_{ij} k_{ij} \frac{\gamma_i \Delta_j}{\tau} + \frac{y_j}{\mu \tau x_j} + \frac{\gamma_i \Delta_j}{\mu \tau x_j})] \\
&= \sum_{j \in \mathcal{A}_i} \gamma_i \Big[\Big(\frac{p_{ij} \gamma_i}{\tau}(\lambda d_{ij} k_{ij} + \frac{1}{\mu x_j}) + \lambda d_{ij} \alpha_{ij} + \frac{y_j}{\mu \tau x_j}\Big)\Delta_j \\
&\quad + (\lambda d_{ij} k_{ij} \frac{\gamma_i}{\tau} + \frac{\gamma_i}{\mu \tau x_j})\Delta_j^2\Big].
\end{aligned} \tag{3.16}$$

For the ease of description, we rewrite it as

$$\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = \sum_{j \in \mathcal{A}_i} \gamma_i (\xi_{ij} \Delta_j + \eta_{ij} \Delta_j^2). \tag{3.17}$$

We can reformulate the CSPP **P1** with the Nash equilibrium definition – no player has the incentive to deviate.

$$\textbf{P2:} \quad \min_{\mathbf{x}, \mathbf{P}} C(\mathbf{P}), \tag{3.18}$$

$$\text{s.t.} \quad \Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \geq 0, \forall i \in \mathcal{N}, \forall \Delta\mathbf{p}, \tag{3.19}$$

(3.9), (3.37) – (3.13).

We use Equation (3.19) to restrict the Nash equilibrium space in stead of using Equation (3.10). In this case, we have reformulated the bi-level optimization problem into a single-level one. However, there is an infinite number of constraints in the problem, because $\Delta\mathbf{p}$ of Equation (3.19) for each i is a vector with continuous elements. Thus we need to furthermore find a way to solve the problem **P2**. We propose a *simple deviation approach*, which can replace Equation (3.19) with a finite number of constraints and make the optimization problem solvable.

3.3.2 Simple Deviation Approach

Before introducing the approach, we first define a special type of deviation called *simple deviation*.

Definition 3.1 (simple deviation)**.** A simple deviation of type i player is a strategy change, where only the probabilities of a pair of pure strategies are changed (one increases and the other decreases by the same amount), while the probabilities of all the other pure strategies remain unchanged. A simple deviation is denoted as a tuple $\langle l, h, \delta \rangle$ with $\delta > 0$, which corresponds to a deviation vector $\Delta\mathbf{p}$, such that $\Delta_l = -\delta$, $\Delta_h = \delta$, and $\Delta_j = 0, \forall j \notin \{l, h\}$.

We can then prove an important property of CSPP as Lemma 3.2 based on *simple deviation*, which is used for simplifying the equilibrium criterion in the derived program **P2**.

Lemma 3.2. *Given a strategy profile* $\mathbf{P}$ *with* $p_{il} > 0$*, type* i *player cannot reduce her charging cost through a unilateral simple deviation from pure strategy* l *to* h *(i.e., reduce* p_{il} *and increase* p_{ih}*), if and only if* $\xi_{ih} \geq \xi_{il}$*.*

Proof. The basic idea to prove this lemma is to derive the charging cost change due to a unilateral simple deviation and analyze it. From Definition 3.1 and Equation (3.17), we can see the charging cost change due to a unilateral simple deviation $\langle l, h, \delta \rangle$ for type i players can be denoted as

$$\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = \gamma_i(\eta_{il} + \eta_{ih})\delta^2 + \gamma_i(\xi_{ih} - \xi_{il})\delta. \tag{3.20}$$

Note that this is a quadratic function of δ. While player i with a nonzero simple deviation $\Delta\mathbf{p} = \langle l, h, \delta \rangle$ cannot reduce her charging cost, we have $p_{il} > 0$ and $\delta \in [0, p_{il}]$; what we need to prove is $\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \geq 0$. From Equation (3.16) we can easily get $\eta_{il} + \eta_{ih} > 0$. As a result, ξ_{ih} has to be no smaller than ξ_{il} to ensure $\Delta C_i(\mathbf{P}, \Delta\mathbf{p})$ to be non-negative for all possible value of δ. We can show this with a discussion in two cases. First, if $\xi_{ih} < \xi_{il}$, there is always some $\delta < \frac{\xi_{il} - \xi_{ih}}{\eta_{il} + \eta_{ih}}$ such that $\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) < 0$. Second, if $\xi_{ih} \geq \xi_{il}$, we can easily see that $\Delta C_i \geq 0$ for all $\delta \geq 0$. Therefore, type i player with $p_{il} > 0$ cannot reduce her charging cost through a simple deviation from pure strategy l to h if and only if $\xi_{ih} \geq \xi_{il}$. □

Lemma 3.3. *If a player cannot reduce her cost by any unilateral simple deviation, then she can neither reduce her cost by any unilateral strategy deviation.*

Proof. Before proving the lemma, we show that an arbitrary unilateral strategy deviation $\Delta\mathbf{p}_i$ for any player i can be decomposed into a number of unilateral simple deviations, thus charging cost change $\Delta\mathbf{p}_i$ can also be decomposed. For simplicity, we denote the unilateral strategy deviation of player i as $\Delta\mathbf{p} = (\Delta_1, ..., \Delta_n)$. For the elements in the vector $\Delta\mathbf{p}$, there must be negative and positive ones, for which we use two sets $\mathcal{L} = \{i | \, i \in \mathcal{N}, \Delta_i < 0\}$ and $\mathcal{H} = \{i | \, i \in \mathcal{N} \Delta_i > 0\}$ to represent respectively. We can see that implement of deviation $\Delta\mathbf{p}$ can be achieved by a number of simple deviations, where each is a deviation from an $l \in \mathcal{L}$ to an $h \in \mathcal{H}$ with the proportion $\delta_{hl} = |\Delta_l| \cdot \frac{\Delta_h}{\sum_{i \in \mathcal{H}} \Delta_i}$.

Consequently, we decompose the change in charging cost due to an arbitrary strategy deviation as in the following.

$$
\begin{aligned}
\frac{\Delta C_i(\mathbf{P},\Delta\mathbf{p})}{\gamma_i} &= \sum_{j\in\mathcal{A}_i}(\xi_{ij}\Delta_j+\eta_{ij}\Delta_j^2) \\
=\sum_{l\in\mathcal{L}}&\Big(\xi_{il}(-\sum_{h\in\mathcal{H}}\delta_{hl})+\eta_{il}(-\sum_{h\in\mathcal{H}}\delta_{hl})^2\Big) \\
&+\sum_{h\in\mathcal{H}}\Big(\xi_{ih}(\sum_{l\in\mathcal{L}}\delta_{hl})+\eta_{ih}(\sum_{l\in\mathcal{L}}\delta_{hl})^2\Big) \\
\geq\sum_{l\in\mathcal{L}}&\Big(\xi_{il}(-\sum_{h\in\mathcal{H}}\delta_{hl})+\eta_{il}(\sum_{h\in\mathcal{H}}\delta_{hl}^2)\Big) \\
&+\sum_{h\in\mathcal{H}}\Big(\xi_{ih}(\sum_{l\in\mathcal{L}}\delta_{hl})+\eta_{ih}(\sum_{l\in\mathcal{L}}\delta_{hl}^2)\Big) \\
=\sum_{l\in\mathcal{L}}&\sum_{h\in\mathcal{H}}(\eta_{il}+\eta_{ih})\delta_{hl}^2+(\xi_{ih}-\xi_{il})\delta_{hl}
\end{aligned}
$$

Note that for the ease of presentation, the cost change is divided by the number of EV users in zone i, i.e., γ_i. As we can see from the above formulations, the charging cost change due to an arbitrary strategy deviation can be compared with the sum of the charging cost change due to the set of simple deviations that equal to the original deviation and it is always no smaller than the latter. With the prerequisite of the lemma, we know that player i cannot reduce his charging cost by any simple deviation, including the set of simple deviations we had as a decomposition of the arbitrary strategy deviation. Referring to Lemma 3.2, we can know $\xi_{ih}\geq\xi_{il}$ is true for all $l\in\mathcal{L}$ and $h\in\mathcal{H}$, i.e., the part to be summed in the right hand side of the above inequality is non-negative. Thus we have proved that $\Delta C_i(\mathbf{P},\Delta\mathbf{p})\geq 0$. Since $\Delta\mathbf{p}$ and i are arbitrary, thus no player can reduce her charging cost by any unilateral strategy deviation while they cannot achieve that with any unilateral simple deviation. □

Proposition 3.4. *A strategy profile* $\mathbf{P}$ *forms a Nash equilibrium if and only if* $\xi_{ih}\geq\xi_{il},\forall i\in\mathcal{N},\forall l,h\in\mathcal{A}_i,p_{il}>0$.

Proof. The proposition is quite straightforward if we follow Lemma 3.2, Lemma 3.3 and the converse direction of Lemma 3.3, which must hold because a simple deviation is a special case of arbitrary strategy deviation. Under the equilibrium definition, no player can decrease the charging cost with arbitrary unilateral strategy deviation ⇔ no player can decrease his charging cost using any unilateral simple deviation $\Leftrightarrow \xi_{il} \leq \xi_{ih}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{ij}>0$. $\square$

Based on Proposition 3.4, we can avoid an infinite number of non-linear constraints as Equation (3.19). With the results from Proposition 3.4, we know that under the equilibrium strategy profile $\mathbf{P}$, there is $\xi_{ih} \geq \xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{il} > 0$, which can be reformulated as $p_{il}\xi_{ih} \geq p_{il}\xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i$. Therefore, we propose OCEAN (Optimizing eleCtric vEhicle chArging statioN placement) in program **P3** to compute the optimal solution of the CSPP instead of using program **P2**.

$$\textbf{P3:} \quad \min_{\mathbf{x},\mathbf{P}} C(\mathbf{P}), \tag{3.21}$$

$$\text{s.t.} \quad p_{il}\xi_{ih} \geq p_{il}\xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, \tag{3.22}$$

$$(3.9), (3.37) – (3.13).$$

The above program is a single-level non-linear optimization problem and can be handled by a standard non-linear optimization solver.

3.3.3 Problem Analysis

An important concept in game theory is the price of anarchy (PoA) [121], which is the ratio between the maximum social cost among different equilibria and the minimum social cost regardless of players' selfish behavior (in other words, assuming the players follow the instruction of a central controller who aims to minimize the social cost). PoA is a concept that measures the worst-case inefficiency of the system caused by the selfish behavior of players. We use S and E to respectively denote the strategy space and Nash equilibrium strategy space of the charging game. They can be formally defined as

$$S = \{\mathbf{P}|\mathbf{P} \text{ satisfies Eqs. } (3.37) – (3.13)\}, \tag{3.23}$$

$$E = \{\mathbf{P}|\mathbf{P} \text{ satisfies Eqs. } (3.37) – (3.13), (3.22)\}. \tag{3.24}$$

Then the definition of PoA is

$$\text{PoA} = \max_{\mathbf{P}\in E} C(\mathbf{P})/Opt, \tag{3.25}$$

where Opt denotes the socially optimal cost assuming that all EVs' charging behavior can be controlled, which is

$$Opt = \max_{\mathbf{P}\in S} C(\mathbf{P}). \tag{3.26}$$

We can prove the theoretical result of PoA as in the following theorem.

Theorem 3.5. *The price of anarchy of the charging game is at most* $\frac{3+\sqrt{5}}{2} \approx 2.618$.

Proof. For the ease of description, we first rewrite the linear cost functions (i.e., travel cost and queuing cost) as $c_e(f_e) = a_e f_e + b_e$ for each congestion element $e \in \mathcal{N} \bigcup \mathcal{R}$. According to Eqs. (3.1) to (3.3), we have

$$a_e = \begin{cases} \lambda d_{ij} k_{ij} \frac{1}{\tau}, & e = \langle i,j\rangle \in \mathcal{R}; \\ \frac{1}{\mu\tau x_i}, & e = i \in \mathcal{N}; \end{cases}$$

$$b_e = \begin{cases} \lambda d_{ij} k_{ij} \alpha_{ij}^0, & e = \langle i,j\rangle \in \mathcal{R}; \\ 0, & e = i \in \mathcal{N}. \end{cases}$$

Obviously, $a_e > 0$ and $b_e \geq 0$. Let $\mathbf{P}$ be a Nash equilibrium strategy, and $\mathbf{P}^*$ be the strategy profile for social optimum. Suppose in a Nash equilibrium, player i deviates by playing the social optimal strategy $\mathbf{p}_i^*$, it follows that

$$\begin{aligned} \sum_{j\in\mathcal{A}_i} p_{ij} \sum_{e\in\mathcal{S}_{ij}} c_e(f_e) &\leq \sum_{j\in\mathcal{A}_i} p_{ij}^* \sum_{e\in\mathcal{S}_{ij}} c_e(f_e^*) \\ &\leq \sum_{j\in\mathcal{A}_i} p_{ij}^* \sum_{e\in\mathcal{S}_{ij}} c_e(f_e + f_e^*). \end{aligned}$$

The first inequality holds since $\mathbf{P}$ forms a Nash equilibrium, thus player i can never decrease his charging cost by unilaterally deviating his own strategy.

Since the above inequality holds for all player i, we have

$$\begin{aligned}
C(\mathbf{P}) &= \sum_{i\in\mathcal{N}} \gamma_i \sum_{j\in\mathcal{A}_i} p_{ij} \sum_{e\in\mathcal{S}_{ij}} c_e(f_e) \\
&\leq \sum_{i\in\mathcal{N}} \gamma_i \sum_{j\in\mathcal{A}_i} p^*_{ij} \sum_{e\in\mathcal{S}_{ij}} c_e(f_e + f^*_e) \\
&= \sum_{i\in\mathcal{N}} \gamma_i \sum_{j\in\mathcal{A}_i} p^*_{ij} \sum_{e\in\mathcal{S}_{ij}} \left[c_e(f^*_e) + a_e f_e \right] \\
&= C(\mathbf{P}^*) + \sum_{i\in\mathcal{N}} \gamma_i \sum_{j\in\mathcal{A}_i} p^*_{ij} \sum_{e\in\mathcal{S}_{ij}} a_e f_e \\
&= C(\mathbf{P}^*) + \sum_{e\in\mathcal{N}\bigcup\mathcal{R}} a_e f_e f^*_e.
\end{aligned}$$

We apply the Cauchy-Schwarz inequality to the last term and get following inequality:

$$\begin{aligned}
\sum_e a_e f_e f^*_e &\leq \sqrt{\sum_e a_e f_e^2} \cdot \sqrt{\sum_e a_e (f^*_e)^2} \\
&\leq \sqrt{\sum_e f_e \cdot (a_e f_e + b_e)} \cdot \sqrt{\sum_e f^*_e \cdot (a_e f^*_e + b_e)} \\
&= \sqrt{C(\mathbf{P})} \cdot \sqrt{C(\mathbf{P}^*)}.
\end{aligned}$$

It follows that

$$\begin{aligned}
& C(\mathbf{P}) \leq C(\mathbf{P}^*) + \sqrt{C(\mathbf{P})} \cdot \sqrt{C(\mathbf{P}^*)} \\
\Rightarrow & \frac{C(\mathbf{P})}{C(\mathbf{P}^*)} \leq 1 + \sqrt{\frac{C(\mathbf{P})}{C(\mathbf{P}^*)}} \\
\Rightarrow & 0 \leq \sqrt{\frac{C(\mathbf{P})}{C(\mathbf{P}^*)}} \leq \frac{1+\sqrt{5}}{2} \quad \text{(by solving } x^2 - x - 1 \leq 0) \\
\Rightarrow & \frac{C(\mathbf{P})}{C(\mathbf{P}^*)} \leq \frac{3+\sqrt{5}}{2} \approx 2.618.
\end{aligned}$$

Thus we can conclude that the PoA is at most around 2.618. Note that the value 2.618 holds for any charging station placement $\mathbf{x}$. Therefore we can rewrite it more accurately as PoA $= \max_{\mathbf{x}} \max_{\mathbf{P}\in E} \frac{C(\mathbf{P})}{Opt}$. □

Furthermore, with the formulation **P3** that we derived in the previous section, we can compute the PoA for a specific setting in practice as follows, which could be much lower than 2.618. Note that when we compute the optimal solution of the charging station placement problem with **P3**, we are actually computing the charging station placement $\mathbf{x}^*$ with the best minimum equilibrium social cost, i.e.,

$$\mathbf{x}^* \in \arg_{\mathbf{x}} \min_{\mathbf{x},\mathbf{P}\in E} C(\mathbf{P}).$$

Then, we can compute the POA for this specific setting based on charging station placement $\mathbf{x}^*$. We compute the maximum equilibrium social cost $\max_{\mathbf{P}\in E} C(\mathbf{P})$ and social optimum Opt respectively with following programs **P4** and **P5**.

$$\begin{aligned}
\textbf{P4:} \quad & \max_{\mathbf{P}} C(\mathbf{P}), && (3.27)\\
\text{s.t.} \quad & \mathbf{x} = \mathbf{x}^*, && (3.28)\\
& (3.37) – (3.13), (3.22). &&\\
\textbf{P5:} \quad & Opt = \min_{\mathbf{P}} C(\mathbf{P}), && (3.29)\\
\text{s.t.} \quad & \mathbf{x} = \mathbf{x}^*, && (3.30)\\
& (3.37) – (3.13). &&
\end{aligned}$$

Note that to compute the optimal social cost without considering the EVs' selfish driving behavior, we eliminate conditions represented by Equation (3.22) in **P5**. As we will show later in the experiment section, the computed POA for specific settings is much smaller than 2.618.

3.3.4 Speeding Up OCEAN

As we can see from the formulation of OCEAN in **P3**, it is a mixed integer non-linear problem and the number of non-linear constraints expressed in Equation (3.22) grows very fast with the number of players and strategies increasing. As a result, OCEAN is unable to handle large-scale real-world problems.

To handle large-scale problems, we compute the optimal solution in two steps by using a heuristic algorithm OCEAN-C (namely OCEAN with Continuous variables), which is shown in Algorithm 1.

Firstly, we relax $\mathbf{x}$ to be continuous variables and solve the optimal solution $\mathbf{x}^*$ of **P3**. Since the number of chargers in $\mathbf{x}^*$ of different zones are not integers, we round $\mathbf{x}^*$ to $\widehat{\mathbf{x}}$. The rounding process is first to take the floor value of each x_i^*, sort the zones according to the $x_i - \lfloor x_i \rfloor$ value descendingly, then set $\widehat{x}_i$ for the top $R = B - \sum_{i \in \mathcal{N}} \lfloor x_i^* \rfloor$ zones as $\lfloor x_i \rfloor + 1$ and otherwise $\lfloor x_i \rfloor$. To compute the optimal solution of CSPP, we set $\mathbf{x}$ as $\widehat{\mathbf{x}}$, the result of which is the output of OCEAN-C. With $\mathbf{x}$ determined, the single level CSPP's runtime sharply decreases.

Algorithm 1: OCEAN-C

```
Relax x to be continuous;
Solve optimal solution x* of P3;
x̂ ← rounded x*;
Compute the optimal solution Obj of P3 with x set as x̂ (refer to Algorithm 2);
return Obj, x̂;
```

Furthermore, we specify the sub-algorithm of OCEAN-C in Algorithm 2, which is designed to compute the equilibrium of the charging game with a given charging station placement. As we can see from **P3**, the problem is non-linear, and the main difficulty comes from distinguishing the employed pure strategies (with using probability > 0) from the abandoned ones (with using probability $= 0$), which results in constraint denoted by Equation (3.22). Then we naturally consider specifying the employed strategies (also named "support") before solving the equilibrium. Following the idea, we design Algorithm 2 to compute the equilibrium, where we first initiate the support manually and gradually expand the support set by carefully comparing the pure strategies until an equilibrium is reached. For a given support set, we use the following program to compute the equilibrium.

$$\textbf{P6:} \quad \min_{\mathbf{P}} C(\mathbf{P}), \tag{3.31}$$

$$\text{s.t.} \quad p_{ij} = 0, \forall i \in \mathcal{N}, \phi_{ij} = 0, \tag{3.32}$$

$$\xi_{ih} \geq \xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, \phi_{ih} = \phi_{il} = 1, \tag{3.33}$$

(3.9), (3.37) – (3.13).

Note that the vector ϕ is an artificial indicator corresponding to the variable $\mathbf{P}$. When $\phi_{ij} = 0$, we force p_{ij} as 0; if $\phi_{ij} = 1$, then $p_{ij} > 0$ and the corresponding pure strategy is in the support. We only compare the ξ value for strategies in the support set to avoid the problem to be infeasible when there is a pure strategy that is not in the support set but its corresponding ξ value is smaller. The problem **P6** is a convex optimization problem with linear constraints that can be solved efficiently.

In Lines 1 – 6 of the algorithm, we initiate the indicator for each pure strategy according to the *basic* charging cost calculated by assuming that only one player uses the corresponding strategy (as in Line 2) and put some of the pure strategies into the support by comparing the *basic* charging cost. Note that the coefficient ψ in Line 6 is to decide the size of the initial support set and derived from practice. *Rule A* and *Rule B* are two criterions for updating the support space. *Rule A* is used to delete the useless strategies and *Rule B* is for adding better pure strategies into the support. When no change is made after checking the two rules, the algorithm terminates with an equilibrium of the charging game.

3.4 Experimental Evaluation

In this section, we run experiments on the real data set from Singapore to evaluate our approach. To compare multiple methods, all experiments were run on the same data set using a $3.4GHz$ Intel processor with $16GB$ of RAM, employing KNITRO (version 9.0.0) for nonlinear programs. The results were averaged over 20 trials.

Algorithm 2: Sub-OCEAN-C

Initiate indicator vector ϕ as $\{0\}$;
Set $f_{ij}^0 = \lambda d_{ij}(\alpha_{ij}^0 + k_{ij})$ and $g_i = \frac{1}{\tau x_i}$ for all roads and charging stations with $x_i > 0$;
for $i \in \mathcal{N}$ **do**
    Set $c_i^{min} = \min_{j \in \mathcal{A}_i} c_{ij}^0 = \min_{j \in \mathcal{A}_i}(f_{ij}^0 + g_j^0)$;
    for $j \in \mathcal{A}_i$ *with* $x_j > 0$ **do**
        if $c_{ij}^0 \leq \varphi c_i^{min}$ **then** Let $\phi_{ij} = 1$;
Set $flag = 1$;
repeat
    Solve problem **P6** and get objective value Obj;
    Set $flag = 0$;
    for $i \in \mathcal{N}$ **do**
        /* ---------- Rule A ---------- */
        for $k \in \mathcal{A}_i$ *with* $\phi_{ik} = 1$ **do**
            if $p_{ik} < 1.0e - 6$ **then**
                Let $\phi_{ik} \leftarrow 0$;
                Set $flag = 1$;
        /* ---------- Rule B ---------- */
        Get a ξ_{ij} with $\phi_{ij} = 1$;
        for $k \in \mathcal{A}_i$ *with* $\phi_{ik} = 0$ **do**
            if $\xi_{ik} < \xi_{ij}$ **then**
                Let $\phi_{ik} \leftarrow 1$;
                Set $flag = 1$;
until $flag = 0$;
return Obj;

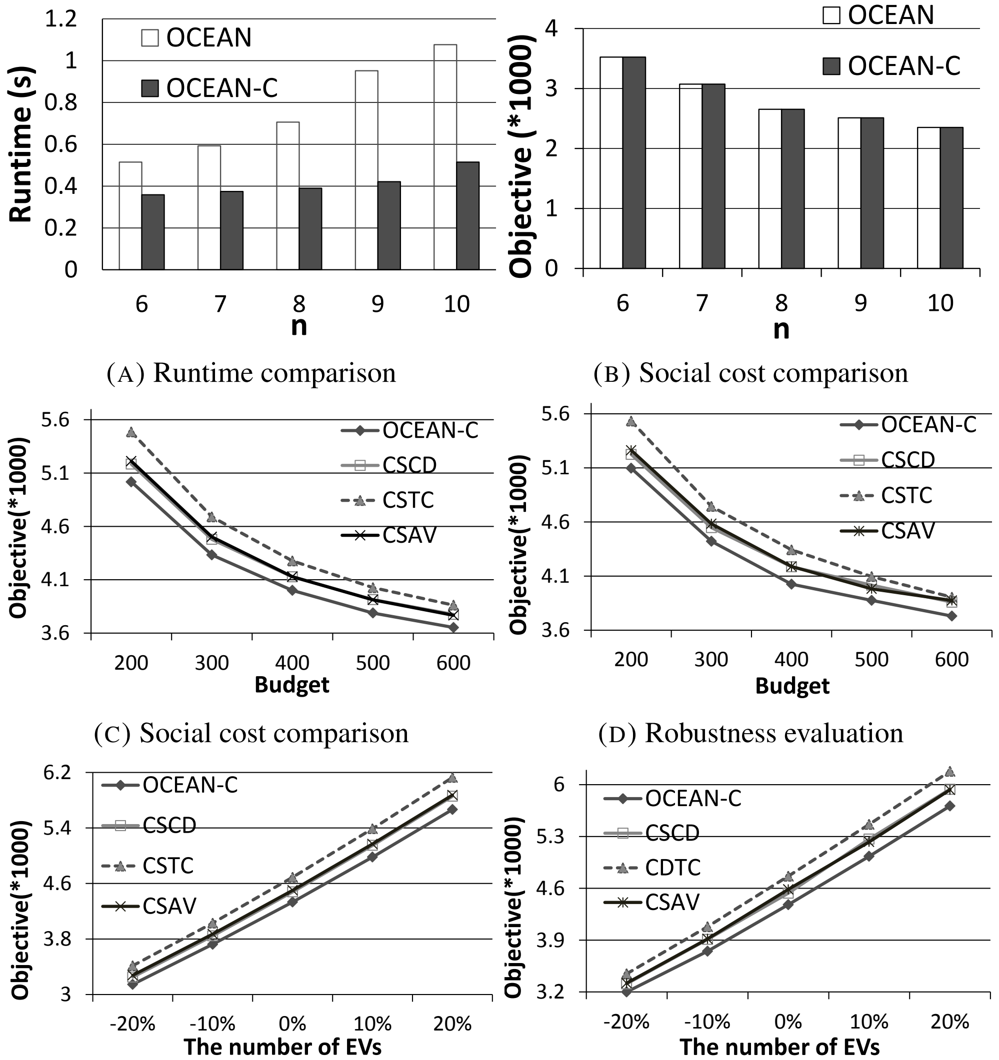
3.4.1 Data Set and Baseline Methods

The population of all motor vehicles in Singapore has reached $969,910$ in the year 2012 according to the statistics in the official websites of Singapore Land Transport Authority (LTA) and Singapore Department of Statistics (DOS). Based on the conventional partition method as shown in Figure 1.1, combined with the accessible graphical and residential distribution data on the websites, we divide Singapore into 23 zones to test our approach. A basic assumption is that the number of vehicles is proportional to the number of residents in each zone. Then we assume that 10% among all the vehicles in Singapore are EVs, 5% of which would need charging in charging stations during peak hours. Using the distance measure tool in Google Maps, the distances between adjacent zones' centers are estimated; a normal congestion α^0_{ij} during the peak hours is taken with the ratio of travel time during peak hours and the distance between zones i and j. The capacity of the roads between any two zones is set to the same value, which means $k_{ij} = 0.01$ for all pairs i and j. We assume averagely 6 EVs can be served in one hour by each charger, i.e., serving rate of chargers is set as $\mu = 6$. The proportion of EVs that charge during peak hours is set as $\frac{1}{\tau} = \frac{1}{10}$. The linear coefficient λ in the travel cost function is fixed at 0.2. Unless otherwise specified, we use the above parameters in all our experiments. We combine some small zones of the 23-zones to generate data of different n (from 6 to 10), so that we can run OCEAN, which has scalability issues, to get the results (both runtime and solution quality) and compare them with OCEAN-C.

To demonstrate the performance of our approach, we compare it with three baseline methods:

- The first baseline method is named CSCD. CSCD assigns the number of chargers to each zone proportional to the number of residential EV users in each zone. Specifically, $x_i \propto \gamma_i$.
- The second baseline method is named CSTC. CSTC assigns the number of chargers in each zone according to the traffic condition as well as the physical distance. Specifically, for each zone i and one of its adjacent zone j, we calculate the reciprocal of $\alpha^0_{ji} d_{ji}$ (intuitively, this value means the difficulty for EV users in zone j to charge in zone i), then sum that value of all adjacent zones together. We decide the number of chargers in zone i as $x_i \propto \sum_{j \in \mathcal{A}_i} 1/(\alpha^0_{ji} d_{ji})$.
- The third baseline method is named CSAV. CSAV assigns the chargers in different zones averagely.

We get the results (the optimal social cost) for each baseline method by first compute the number of chargers in each zone according to the principals described above, then compute the EV users' charging activity equilibrium and the resulted social cost. The program is the same as program **P3** but $\mathbf{x}$ is fixed rather than a variable.



(A) Runtime comparison

(B) Social cost comparison

(C) Social cost comparison

(D) Robustness evaluation

(E) Social cost comparison

(F) Robustness evaluation

FIGURE 3.3: Compare OCEAN-C with OCEAN and baselines. Figures (a) & (b) are the runtime and optimal objective value of algorithm OCEAN and OCEAN-C when problem size increases; Figure (c) is optimal objective values of OCEAN-C and baseline methods when the budget increases, while Figure (d) is the corresponding results with human behavior uncertainty; Figure (e) and f are results with different number of EV drivers in the charging game.

3.4.2 Performance Evaluation

In this section, we present the experimental results and discuss them in detail.

3.4.2.1 OCEAN-C VS. OCEAN

We combine some small zones of the 23-zone division (shown in Figure 1.1) to get smaller zone divisions (n changes from 6 to 10) because OCEAN cannot handle large-scale problems. The budget of the total number of chargers is set as 300 in the experiments. In Figures 3.3a, the runtime performance of OCEAN and OCEAN-C are compared with bars. We can see that OCEAN-C always saves runtime comparing to OCEAN. Moreover, when the problem scale grows, the runtime of OCEAN increases faster than that of OCEAN-C, which indicates that OCEAN-C is much more time-efficient than OCEAN. When we look at the solution quality (i.e., the optimal social cost) depicted in 3.3b, we can find that OCEAN-C can save runtime without seriously sacrificing the solution quality because the minor difference in social cost of both approaches is invisible when expressed with the bars. Therefore, we use OCEAN-C as a substitute approach for OCEAN in following experiments.

3.4.2.2 OCEAN-C VS. Baseline Methods

We compare our approach OCEAN-C with three baseline methods when the number of zones n is set as 23. As we can see from Figure 3.3c, when the budget is increasing (from 200 to 600), the optimal objective value of all approaches keeps going down, because more resources usually mean better service and customer convenience. Nevertheless, our approach outperforms all of them and achieves minimal social cost. In Figure 3.3e, the results of changing the number of EV users are depicted. We can see that when the number of EV users is more, the minimal social cost is higher because they have more influence on the traffic congestion and also the queuing condition in charging stations. In this case, our approach that takes into account the EV users' s-trategic behaviors can significantly decrease the social cost. In conclusion, OCEAN-C outperforms the baseline methods.

3.4.2.3 Robustness Evaluation

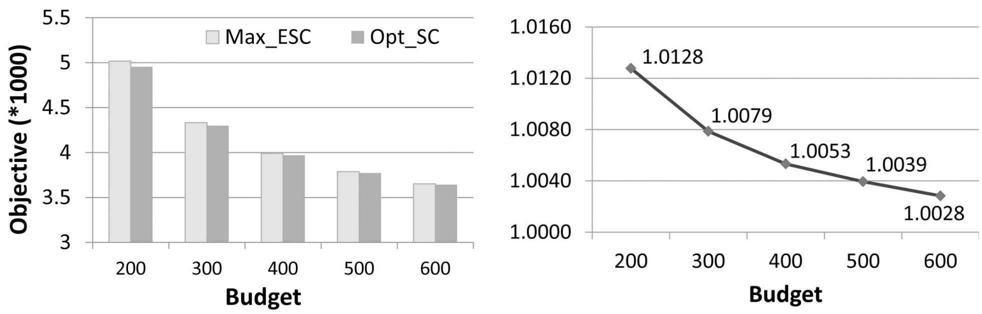
We then evaluate the robustness of our approach and compare its performance with the baseline methods regarding the EV users' limited rationality. We assume that EV users are fully informative and rational in the problem model. While people might be able to learn the equilibrium in repeated charging activities, there can be some special cases that change their activity in practice. For example, they might need to deal with a special thing or meet someone, which may result in strategy deviation. We assume that there are part of EV users deviate their charging activities from equilibrium, This proportion is set as 10% for each zone, i.e., we compute the social cost again with the 90% of EV users following the equilibrium and 10% of them choose a charging strategy from their strategy space randomly. In Figures 3.3d and 3.3f, we present the robustness test results for all approaches in consideration of a different budget and a different number of EV users respectively. The number of zones is set $n = 23$. Under comparison with Figures 3.3c and 3.3e, we can see that the EV users' deviation from equilibrium can cause more social cost. However, our approach OCEAN-C can keep the superiority compared to the baseline methods.

3.4.2.4 EVs Charging in Remote Zones

When we formulate the charging game previously, we made an assumption that the EV users only charge in adjacent zones (including their residential zone). To prove that this assumption is realistic and reasonable, we use experiments to show that almost all EV users only charge in adjacent zones even when they are allowed to charge further because the latter usually results in higher charging cost. We relax the assumption for EV users in zone i by allowing them to charge in a neighbor zone of its adjacent zones (but not in $\mathcal{A}_i$), which is name two-stop remote zones. We compare the results of the original model and the new one under the same data set. It turns out that the social cost increases slightly, but the change is less than 0.001, which is negligible compared to the original optimal social cost at about 4000). Moreover, the EV users seldom use the two-stop strategies. As a result, we can see that the assumption of charging only in adjacent zones is realistic and reasonable.

3.4.2.5 Experimental Results of PoA

We conduct experiments based on optimal charging station placement derived from OCEAN-C and the experiment set with $n = 23$. The coefficient φ used for initiating the support set in Algorithm 2 Line 6 is set as 1.5. Actually, the coefficient can vary in a big range and still work. When it is getting smaller, the number of iterations of solving problem **P6** can increase; and when it is too large, it is possible that the problem becomes infeasible. We select 1.5 as φ value in this set of experiment. The maximum equilibrium social cost and the minimum social cost without consideration of EVs' selfish charging behavior are respectively computed with programs **P4** and **P5**.



(A) Social cost (B) PoA under different budgets

FIGURE 3.4: (a) Compare maximum equilibrium social cost and social optimal; (b) Trend of PoA under different budgets

As we can see from following Figure 3.4a, there is a small difference between the maximum equilibrium social cost and optimal social cost respectively depicted by the "Max_ESC" and "Min_SC" bars. We can refer to Figure 3.4b for the trend of PoA w.r.t. the budget. From the figure, we can see that when the amount of social resource (i.e., the budget for charging station construction here) increases, the inefficiency of the charging system caused by selfish behavior is becoming smaller.

3.5 Extension on Senior Center Placement and Management

Population aging has been a common phenomenon in almost every country in the world. Since 1950, the number of people aged above 50 has tripled, and the combined senior and geriatric population will reach 2.1 billion and exceed 16% (as shown in Figure 3.5) of the world population by 2050 according to studies [122–124]. Accompany with the population aging is the raising challenges for the society to ensure life quality of the elderly because frailty is a common condition in later older age [125]. Both mental and physical health issues of senior citizens require intensive attention from the family and the government. A senior center is a type of community center that is promising to provide an environment for older adults to fulfill many of their social, physical, emotional and intellectual needs.

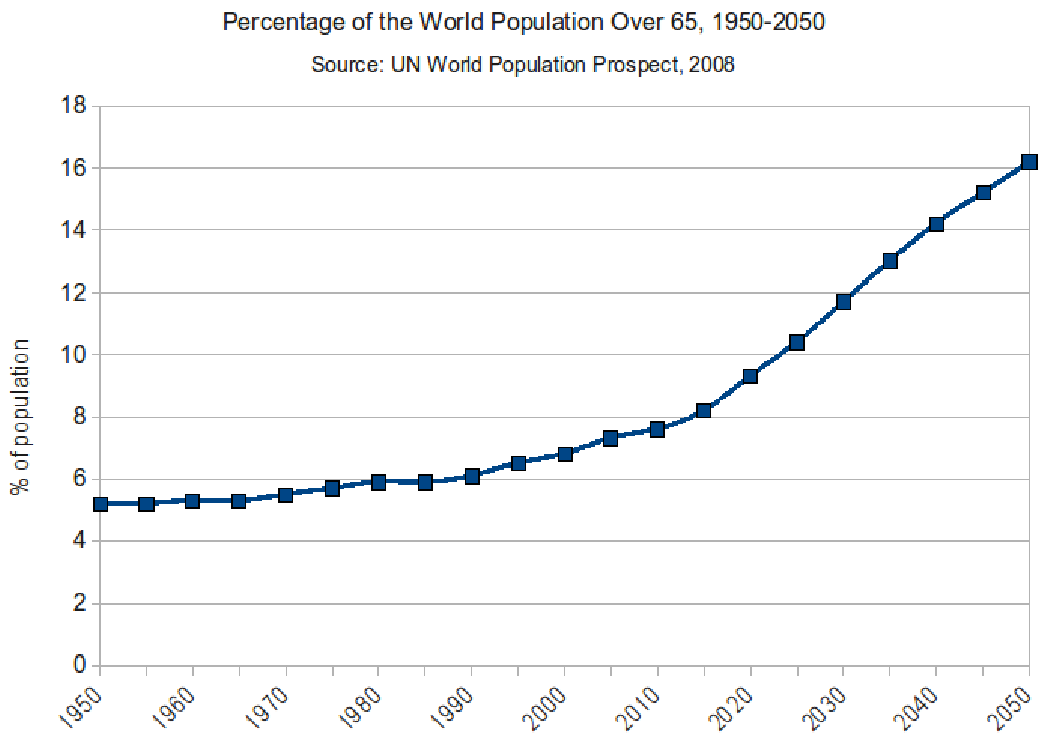


FIGURE 3.5: World population over 65

Multi-purpose senior centers have been proven to be important since their advent in 1943 because they provide an ideal environment for promoting positive and successful aging activities [126–129]. As the older persons with no jobs are usually hard to get accompanied and occupied in daily life, especially during working time, loneliness and depression are becoming common among them. Senior centers are an entry point to an array of services that can satisfy their self-sufficiency. Senior centers can provide a variety of opportunities for the elderly to get access to health, education, volunteer, recreation and other social activities, such that their dignity can be enhanced, they can prove their independence and they will be encouraged in community involvement. Moreover, senior centers are able to provide an environment, where they can build a social connection with other people with similar experience and sense of value. Therefore, aging people are provided with the opportunity to get social interaction and friendship in senior centers, which can help them to regain feelings of self-worth and counter the social isolation and loneliness that can threaten their mental and even physical health.

3.5.1 Background

Although there have been a number of works studying the importance and operation mode etc. of senior centers, there is a gap in analyzing their allocation and placement. Senior centers have been constructed in a number of cities in the world. However, there is a gap between requirement and support. For example, in Singapore, there is a network of about 60 elder-care day centers providing day services such as day care and day rehabilitation services. This network of centers has a total capacity to serve 2,800 seniors [130]. However, this is far from enough because the population aging 65 and above of that year in Singapore had exceeded 560 thousand [131]. The aging phenomenon in Singapore is keeping more and more apparent in recent years as shown in Figure 3.6. To meet the demand from the elderly, the government has been increasing the investment in senior centers' construction.

The location of senior centers is an important topic because it involves multiple factors that can influence their operating performance. There are mainly three factors in the senior center allocation problem.

1. The position of senior centers is restricted by the existing city structure and architectures. As a result, the location to settle a senior center and the size of it are both dependent on the specific scenario. The construction cost (especially for the land area) should also be in consideration.

2. Senior centers are aimed at the aging people, and those population usually have difficulty in moving around. Part of the senior center visitors are even disabled or relying on wheelchairs, and they would require family members to send them to

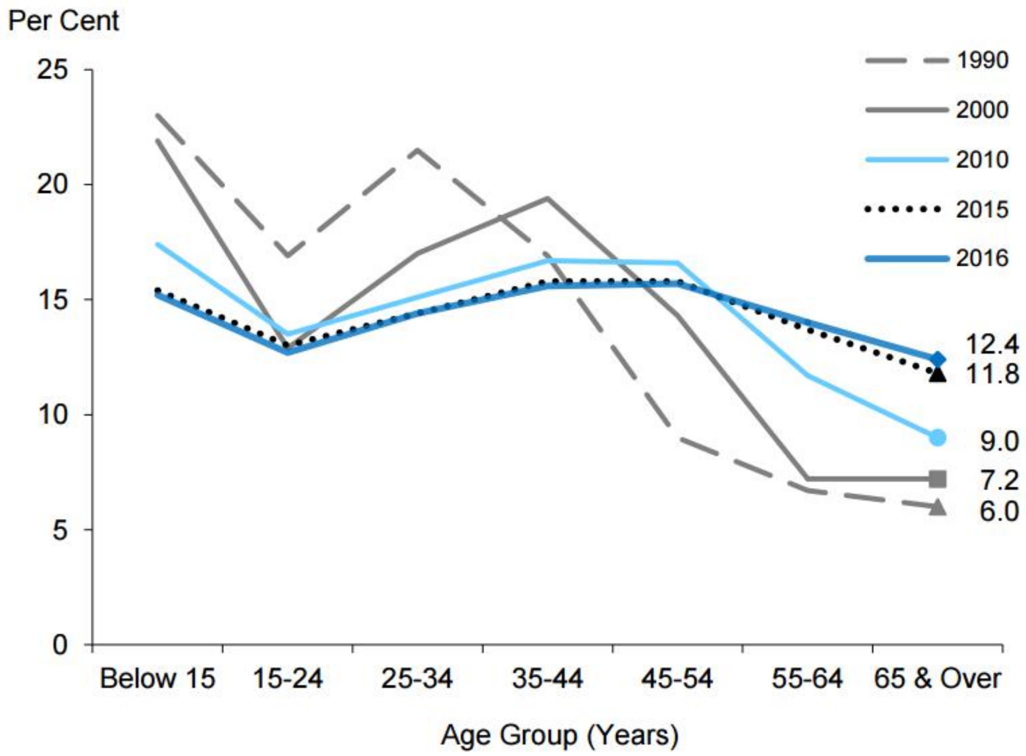


FIGURE 3.6: Age groups of citizens in Singapore from 1990 to 2016

and pick them up from senior centers. Thus the senior centers are expected to be close to the residential area of the aging people.

3. Senior centers need workers and volunteers to help the elderly to deal with all kinds of issues, to take care of some of them, and also contribute in communication with them such that their feeling of loneliness can be degraded, which is extremely important for their mental and physical health.

4. The demand from the citizens will change with time, but it is predictable according to the population statistic data.

3.5.2 Related Work

There have been many works that study the allocation problem of service centers or healthcare facilities. For example, Gupta *et al.* [132] proposed a fuzzy C-Means clustering and particle swarm optimization based scheme to optimally locate service centers in a country's rural regions. Basu *et al.* [133] built an optimization model and designed novel techniques to maximize the health care coverage to deal with the growing health needs in expanding cities. Zhang *et al.* [134] proposed a genetic algorithm based multi-objective optimization (MOO) approach to yield a set of Pareto solutions, which can be used to balance the trade-off between the conflict objectives of health-care facility construction, i.e., the facility coverage and the financial investment. However, none of these approaches can be applied in the senior center allocation problem because they all ignored the impact from the target visitors and there is no consideration of the changing demand.

To fill the gap in existing research and support active living of the elderly, we can extend the research on electric vehicle charging station placement to provide solutions for senior center placement and management respectively. Similarly, the location of senior centers is constricted by many factors and it has a limited degree of freedom, we can assume that the candidate positions are screened previously. Thus we only need to decide which of the candidate locations should be selected to build senior centers and their corresponding size. Without losing generality, we can set the service rate (i.e., the proportion of aging people that can be served by the senior centers) as the objective to maximize, meanwhile, the limited financial cost is considered with constraints. This is flexible because the object can be changed according to the realistic scenario and different goal. For example, the government can also set the satisfactory level of the elderly as the primary goal. The serviceability of senior centers, on one hand, is decided by the center size and supporting facilities; on the other hand, is influenced by support from the public, especially the volunteers. Volunteers are very common and welcome in the aging population because they can bring care and pleasure while workers in senior centers focus more on daily operation tasks. The senior center, as a special kind of resource, is different from charging stations in its serviceability. For charging stations, we can easily see that fewer customers are always better (from the perspective of EV drivers). However, senior centers can provide better service for the elderly when the number of visitors is at a suitable range. If there are few seniors in the center, they would feel lonely; if there are too many, they might feel crowded and uncomfortable. Regarding this feature, we would employ a piecewise function to model the service quality of senior centers. The senior center placement problem can be formulated as an optimization

TABLE 3.2: The notations referred in this paper and the corresponding meaning

Notation	Meaning
N	The set of senior center candidates
x_j	The capacity of senior center to be built at position $j \in N$
w_j	The number of volunteers that senior center j can attract
$C_j(x_j)$	The construction cost of senior center j with capacity x_j
c_j^0	The basement cost of building senior center at position j
c_j	The unit price (regarding to capacity) of building senior center at position j
M	The set of senior groups, where each group of seniors are treated identically
s_i	The number of senior people in group i
α_{ij}	The convenience parameter for senior people in group i going to senior center j
p_{ij}	The proportion of senior people in group i to visit senior center j
h_i	The threshold, when $\alpha_{ij} \geq h_i$, the senior people in group i will visit senior center j
B	The financial budget for senior center construction
$\|T\|$	The future period in years to consider for the performance of senior centers, $T = \{1, ..., \|T\|\}$
s_i^k	The number of senior people in group i after k years, $k \in T$

with consideration of the elderly's choices and solved with similar techniques as we used in above-mentioned works.

3.5.3 Problem Formulation

In this section, we formulate the senior center allocation problem as an optimization problem. Since the position of senior centers is restricted by many factors and it has a limited degree of freedom, we assume that the candidate positions can be screened previously. Thus we only need to decide which of the candidate locations should be selected to build senior centers and their corresponding size. Without losing generality, we set the service rate as the objective to maximize, meanwhile the limited financial cost is considered with constraints. This is flexible because the objective can be changed according to the realistic scenario and different goal. For clarity and ease of understanding, we provide a notation table to show all the notations and their meaning with Table 3.2.

We use a set $N = \{1, 2, ..., |N|\}$ to denote the set of senior center location candidates. The variable to decide in this problem is $\{x_1, x_2, ..., x_{|N|}\}$ where $x_j \geq 0, \forall j \in N$, i.e., the capacity of the senior center (i.e., the number of senior people can be served) built in location j when $x_j > 0$. Note that when x_j is 0, it indicates that this candidate site is not selected and no senior center will be built here. Each senior center candidate j is potential to get w_j volunteers to help the elderly in the senior center, which is dependent on the position of j and the residential condition around it[1]. We consider the cost of hiring full/part-time helpers to be linearly proportional to the capacity of the senior center. Therefore, the construction of senior center j with capacity x_j can be denoted as function

$$C_j(x_j) = \begin{cases} c_j^0 + c_j x_j, & x_j > 0 \\ 0, & x_j = 0 \end{cases} \tag{3.34}$$

where c_j^0 denotes the basement cost of building senior center at candidate j, which is decided by its properties. The coefficient c_j represent the unit price coming from land area, hiring and management cost.

The senior population can be divided into a number of groups according to their residential address because they are close to each other and have the same access convenience to the same position. The senior population groups are represented with a set $M = \{1, 2, ..., |M|\}$, we use s_i to denote the number of aging people in each group i and they are treated identically.

The senior citizens in each group i have a convenience parameter for accessing a senior center located at position candidate j, and we denote it as α_{ij}, whose value is in $[0, 1]$. With a higher α_{ij} value, the senior in group i have a higher willingness to visit location j (merely due to the travel distance or also considering the convenience of their family that sends them to the senior center). We assume this parameter can be obtained from surveys and investigations etc. The action of senior citizens is denoted with a series of binary variables p_{ij}, where $p_{ij} \in [0, 1]$ is the proportion of senior people from group i and go to senior center j. Thus we have $\sum_j p_{ij} \leq 1$ for the group i of senior people. Note that $\sum_j p_{ij}$ can be less than 1, which means some senior people may prefer to stay at home while they cannot find an ideal senior center to visit.

[1] Such information is assumed to be obtained in a preliminary study in candidate selection.

Similarly, we can get $\sum_i s_i p_{ij} \leq x_j$, which means a senior center's capacity is a limitation on the maximum number of visitors. The number of volunteers for each senior center is treated as a factor that influences the service quality in the center. Specifically, we define the service quality of senior center j as $w_j / \sum_i s_i p_{ij}$, i.e., the average number of volunteers serving for each senior visitor. We set a service quality threshold Q for all the senior centers to ensure that the senior can always be comfortable and satisfied in the senior centers.

Sequentially, the senior center allocation problem can then formulated as the following optimization problem **P1**. Equation(3.35) is the objective set as the total number of served seniors. Equation(4.18) denotes the financial budget for senior center construction. Note that B is the budget for financial investment. Equations(3.37) to (3.40) are the constraints for variables $\{p_{ij}\}$ and $\{x_i\}$.

$$\textbf{P1:} \quad \max \sum_i s_i \sum_j p_{ij} \tag{3.35}$$

$$s.t. \quad \sum_j C_j(x_j) \leq B \tag{3.36}$$

$$\sum_j p_{ij} \leq 1, \forall i \in M \tag{3.37}$$

$$\sum_i s_i p_{ij} \leq x_i, \forall j \in N \tag{3.38}$$

$$\frac{w_j}{\sum_i s_i p_{ij}} \geq Q, \forall j \in N \tag{3.39}$$

$$p_{ij} \in [0, 1], \forall i \in M, j \in N \tag{3.40}$$

$$x_i \geq 0, \forall i \in N \tag{3.41}$$

3.5.3.1 The Senior's Choice

As we can see from the formulation **P1**, a key point for solving this problem is to decide the proportion of each group of seniors that visit different senior centers. Intuitively, the senior would prefer senior centers with higher convenience factor, i.e., α_{ij} value. However, the situation that the senior center j gets full can happen and in this case, they would turn to their second choice. Thus, we set a threshold h_i for each group of senior people, which means that they can visit any senior center with $\alpha_{ij} \geq h_i$. Under this circumstances, we can modify Equation(3.40) as following.

$$p_{ij} \in [0,1], \text{if } \alpha_{ij} \geq h_i, \tag{3.42}$$

$$p_{ij} = 0, \text{if } \alpha_{ij} < h_i, \tag{3.43}$$

3.5.3.2 Regarding Changing Demand

As the population structure of a city can be derived from demographic census, we can anticipate the change in senior population in the future years. While construction of senior centers is costly and usually irreversible, the investor or government would prefer to look into the long-term result and make it beneficial for as long as possible. For the senior center allocation problem, we assume the policy aims to achieve the best result in the future $|T|$ years.

First, we denote the expected number of senior people of group i after k years as $s_i^k, \forall k \in T = \{1, ..., |T|\}$. Considering that the construction could take some time, we ignore the situation of the current year. Similarly, we can also predict the number of volunteers that each senior center can attract and denote it as w_j^k for each $j \in N$ and $k \in T$.

Then the senior center allocation problem can be reformed as P2 to maximize the total number of served seniors in $|T|$ years.

$$\textbf{P2:} \quad \max \sum_{k \in T} \sum_{i} s_i^k \sum_{j} p_{ij}^k \tag{3.44}$$

$$s.t. \quad \sum_{j} C_j(x_j) \leq B \tag{3.45}$$

$$\sum_{j} p_{ij}^k \leq 1, \forall i \in N, k \in T \tag{3.46}$$

$$\sum_{i} s_i^k p_{ij}^k \leq x_j, \forall j \in N, k \in T \tag{3.47}$$

$$\frac{w_j^k}{\sum_i s_i^k p_{ij}^k} \geq Q, \forall j \in N, k \in T \tag{3.48}$$

$$p_{ij}^k \in [0, 1], \forall i \in M, j \in N \tag{3.49}$$

$$p_{ij}^k = 0, \forall \alpha_{ij} < h_i \tag{3.50}$$

$$x_i \geq 0, \forall i \in N \tag{3.51}$$

3.5.4 Algorithm

To solve problem **P2**, which is a mixed integer non-linear optimization problem, we can first update value of α_{ij}, such that $\alpha'_{ij} = 1$ for $\alpha_{ij} \geq h_i$ and otherwise $\alpha'_{ij} = 0$. Correspondingly, a number of variables p_{ij} with $\alpha'_{ij} = 0$ can be eliminated and we can represent Equations(16) and (17) as Equation(3.52).

$$0 \leq p_{ij}^k \alpha_{ij} \leq \alpha_{ij} \tag{3.52}$$

Then the problem can be solved with existing solvers to find the optimal senior center allocation plan.

3.5.5 Conclusion

To employ our approach in the real-world scenario, there is need to prepare sufficient preliminary work. As future work, we will study how to scientifically evaluate all the necessary parameters for computing the best solution. Specifically, our future work mainly includes following two aspects.

1. Senior and volunteer population prediction with current population distribution and historical evolution data. Techniques like data mining are considered to be employed in such studies.

2. More detailed human behavior study on how a family would choose a senior center. The decision can be affected by a number of complicated and interacting factors, including but not limited to the senior center's charge, popularity, equipment and service quality, the senior's health condition (whether and how far he/she can go out alone) and so on.

To optimally allocate the senior centers for the best of their performance and the citizens' convenience, we delicately analyze the problem and build a realistic model in this work. The senior's choice on which senior center to use is considered according to their access convenience. Meanwhile, our model can handle the long-term objective, i.e., considering the predictable change of the elderly distribution in the future years and maximizing the total number of served seniors. We would further work on this problem.

3.6 Summary

The key contributions of this chapter include: (1) a realistic model for the CSPP in cities like Singapore considering the interactions among charging station placement, EV drivers' charging activities, traffic congestion and queuing time: (2) an equivalent single-level CSPP of the bi-level CSPP optimization problem obtained through exploiting the structure of the charging game; (3) an effective heuristic approach that can speed up the mixed integer CSPP with a large amount of non-linear constraints; (4) theoretical analysis of PoA and corresponding experiments for the charging game; (5) experiments results based on real data from Singapore, which show that our approach solves an effective allocation of charging stations and outperforms baselines.

There are a few aspects from which we can further improve this piece of work. First, we adopt mixed strategy Nash equilibrium as the solution concept for the charging game, where each player is individually taken into consideration in the equilibrium. However, when the EV driver population is large enough, the influence of individuals would be significantly smaller. We might need to consider other solution concepts for this kind of problem when the EV population grows large. Second, based on the real map of Singapore, we make assumptions on some unavailable parameters (e.g., the number of EV drivers in each zone). In the future, when more real data is available, we might need to carefully proceed the data to get the parameters for the real-world application. Last, the construction of charging stations is quite costly, and thus it is unrealistic to reverse the placement. However, as the city develops, different areas of the city might have different speed in population change. Further study in charging station management is required for satisfying the city development. In the next chapter, we present the work on the pricing problem for charging station management.

Chapter 4

Optimal Pricing for Efficient Electric Vehicle Charging Station Management

Electric Vehicles (EVs) are welcoming a rapid development along with the progress of relevant technologies in recent years. As an eco-friendly substitute for the traditional fuel-engined vehicle, EV is seen as a promising solution to the ever devastating energy crisis and environmental pollution around the globe, thus has drawn increasing attention from the public, markets, decision-makers, and academia. Many countries and cities have proposed plans to promote EV usage or have been preparing to do so, providing a foreseeable vision that EV will become the major vehicle of the private transportation sector in the near future [135]. Notwithstanding the progress, challenges still remain. Limited battery capacity and long charging time, probably the most widely complained disadvantages, raise mileage anxiety and largely impair EV users' driving experience. As a result, charging convenience has become a top concern affecting potential users' choice between EV and traditional fuel-engined vehicle. Specialized EV charging stations, which provide a charging speed more than 10 times faster than domestic charging, are therefore critical to the successful promotion of EV.

To adapt to the urban structure change as well as varying charging demand, a practical solution as we propose in this chapter is to leverage the charging price to readjust

EV users' charging behavior and improve the efficiency of the charging network. Compared with placement, pricing is easily and immediately implementable without additional cost or waste of resources. Dynamic pricing schemes adapt to either long-term changes of travel demand caused by residential movements or short-term variances between peak and non-peak time and serve as a flexible complement to existing charging station placement. Our goal is to optimize the pricing scheme to optimize the efficiency of charging stations, i.e., to minimize the additional cost caused by EV users' charging behavior, which is referred as the social cost. There have been some works leveraging dynamic pricing to improve the efficiency of public transportation systems, such as taxi systems [3, 4]. Some works have particularly focused on real-time pricing and charge-discharge policy for EV management [5, 6]. However, their aim is just to balance electricity load in power grids, while traffic condition is not in their consideration. Moreover, their method cannot be incorporated with trivial modifications because the traffic condition deeply relates to EV users' self-interested charing behavior associated with a graph-based road network, which is all absent from existing work.

In this chapter, we take a game-theoretic perspective and build the problem on a non-atomic congestion game played by EV users. The model incorporates the following key features: 1) EV users' self-interested charging behavior that they strategically choose the best charging plan (i.e., where to charge and how to reach the charge station) to minimize their costs including charging fees, traveling time, and queuing time; 2) EV users' traffic pattern with complex spatial variances; 3) Traffic congestion in the road network that is affected by both the EVs and other external vehicles; and 4) A budget constraint that ensures sufficient income to support a sustainable operation of the charging network. Using this model, we formulate the EV charging station pricing problem as a mixed integer non-convex optimization problem, and propose a scalable algorithm to solve the problem, in particular, to deal with the large strategy space of the EVs. Experiments on both mock and real data are also conducted, which show scalability of our algorithm as well as our solution's significant improvement in social cost over existing approaches. A concrete instance is also used to visualize the difference between our approach and existing approaches.

4.1 Motivation

Singapore is a city-country with a vehicle ownership of around $970,000$ on its small territory of $720\ km^2$. As a highly developed metropolis with open attitudes toward cutting-edge technologies, yet a country with limited natural resource and energy supply, Singapore is actively seeking the possibility of mass adoption of EVs to support its sustainable development. Ever since 2011, its authorities have started an EV test-bed to study the feasibility of EVs on its road. More recently, in the Government's sustainable blueprint to guide the country's development over the next 15 years launched in 2014, Singapore has even planned to lead an EV-sharing project to make the new technology even more convenient and environmentally-friendly.

Indeed, the relatively short driving distances on the small territory and the advanced power grid of Singapore make EVs a good option for this city. However, there are also many difficulties that require every step taken to be carefully planned. Because of the land scarcity and the fact that roads have already taken up 12 percent of Singapore's total land area, there is limited room for further expansion of Singapore's road network. This leaves Singapore a very high road density of $4.8km/km^2$ and a transportation system that is highly sensitive to any changes to the current transportation mode. Besides, Singapore is undergoing a rapid change in the residential pattern along with its continuing development. As shown in Figure 1.2, population growth varies significantly among major residential zones of Singapore, indicating similar significant changes in residential traffic pattern. A sustainable plan, therefore, needs to be compatible with the current system while adaptable to future changes, to ensure a smooth transition toward the new EV-led transportation mode. This motivates our work and offers us a concrete study case.

4.2 Preliminary

In this section, we introduce some notations and definitions that will be helpful to scenario visualization and be used in formulating the problem.

4.2.1 Notations

Considering the residential distribution of the studied city (e.g., Singapore), we divide the region to be analyzed (whole or part of the city) into a set $\mathcal{Z}$ of zones. There are roads linking the zones. Without loss of generality, we assume that there is at most one link between a pair of zones representing the average connectivity between them, and denote the set of links as $\mathcal{E}$ and the road network as a graph $\mathcal{G} = (\mathcal{Z}, \mathcal{E})$. In each zone $i \in \mathcal{Z}$, there are γ_i EVs owned by the residents who have some chance to charge in the charging stations. The γ_i EVs are furthermore classified into K_i groups according to their travel patterns, i.e., their daily travel routine as a set of most frequently visited zones. Each travel pattern is a set of connected zones that they visit daily. We denote by γ_{ij} the number of EVs in each group, and by $\mathcal{P}_{ij}$ their pattern for $j \in \mathcal{K}_i = \{1, \ldots, K_i\}$. The union of all patterns is denoted by $\mathcal{P} = \bigcup_{i \in \mathcal{Z}, j \in \mathcal{K}_i} \mathcal{P}_{ij}$.

Given the number τ_i (≥ 0) of chargers in each zone i, our goal is to calculate the charging rate (i.e., per unit electricity price) x_i to be set at each zone, such that the social cost (to be defined later) is minimized. Accordingly, we denote the set of feasible price as $\mathcal{X}$.

4.2.2 Factors Affecting EVs' Decision

EVs make decisions about where to charge and how to reach the charging zone according to the estimated charging cost, which consists of three parts: charging fee, travel cost and queuing cost. The charging fee is the variable to be optimized in this work. In the following, we introduce the definition of travel cost and queuing cost.

Travel Cost. There are furthermore two kinds of travel cost: 1) *cost on link*, i.e., cost for travelling between zones, and 2) *cost on node*, i.e., the cost of traveling within the zone where the EV charges. The cost on a link depends on the length and the traffic congestion level of this link. Generally, more vehicles on the road result in higher congestion level, and larger road capacity leads to lower congestion. We adopt a widely used linear model of traffic congestion taken as the ratio of the traffic flow to the road capacity (both in the number of vehicles). Thus, for a link $(i, i') \in \mathcal{E}$, given the length

$d_{ii'}$, the capacity $C_{ii'}$ and traffic flow $f^0_{ii'} + f_{ii'}$ (we distinguish flow of EVs heading for charging, i.e., $f_{ii'}$, with flow of other vehicle, i.e., $f^0_{ii'}$, which is assumed to be constant), the traffic congestion $\alpha_{ii'}$ of link (i, i') is presented in Equation (4.1) [116]. Travel cost on (i, i') is defined as a function of its length and traffic congestion as Equation (4.2).

$$\alpha_{ii'} = \frac{f^0_{ii'} + f_{ii'}}{C_{ii'}} \tag{4.1}$$

$$t_{ii'} = d_{ii'}\alpha_{ii'} \tag{4.2}$$

Meanwhile, when an EV chooses to charge in a zone i, there is extra travel cost, i.e., cost on node, as she drives off the main road to access the charging station within zone i. Considering that a zone includes more internal roads (than in between two zones) and that the EVs coming for charging does not have to traverse all of them, we add a discount factor ζ to denote the EVs' influence on congestion in the zone. In this case, a similar function as Equation (4.2) is used for the extra travel cost in the charging zone:

$$t_i = d_i\frac{f^0_i + \zeta f_i}{C_i}, i \in \mathcal{Z}, \tag{4.3}$$

where d_i denotes the radius of the zone, f^0_i is the normal traffic amount, f_i is the total number of EVs that choose to charge in zone i and C_i is the capacity of the zone regarding all its travel network.

Queuing Cost. The queuing cost depends on the number of chargers in the charging station and the number of EVs that come to the charging station. We use a linear model to denote the relationship between them as Equation (4.4). Recalling that τ_i denotes the number of chargers in zone i, let q_i be the queuing cost and f_i be the number of EVs charging there, then we have

$$q_i = \frac{f_i}{\tau_i}, i \in \mathcal{Z}. \tag{4.4}$$

4.3 EVs' Charging Behavior

From a game-theoretic perspective [119], when we optimize charging station management, we need to take into account the strategic behavior of EV owners. Namely, they are self-interested and profit-driven, such that they will respond to our pricing with the best charging strategy to minimize their charging cost. Next, we explicitly explain the charging game - the model of EVs' charging behavior. To distinguish a zone that EVs reside in from one EVs charge in, we use i and z respectively to denote a zone in the follows.

4.3.1 Charging Strategy

Each EV chooses a pure charging strategy and the strategies of the EVs in the same zone of the same travel pattern form a distribution over their strategy space. A pure charging strategy is to choose a zone with charging stations installed, and an additional travel path from a zone on her daily routine to the charging zone and back if the chosen zone is not in the EV's travel pattern. Thus a pure charging strategy can be denoted as a tuple s with

$$\mathrm{s} = \begin{cases} \{z\}, z \in \mathcal{P}_{ij} \\ \left\{ z', (z, z'), (z', z) \,\middle|\, z \in \mathcal{P}_{ij}, z' \in \mathcal{Z} \setminus \mathcal{P}_{ij}, \right\} \end{cases}, \tag{4.5}$$

where both (z, z') and (z', z) are in set $\mathcal{E}$. Note that by Equation (4.5), we only consider EVs' charging zones inside or adjacent to zones in their travel pattern and assume that they do not charge in farther places. This is because the distance to those places is usually much farther than a single hop, thus causing higher travel cost and is unlikely to happen in reality. Experimental results in Section 4.5.3 verify that this assumption is reasonable. We then use $\mathcal{S}_{ij}$ to denote the strategy space for the EVs in zone i of travel pattern j. Furthermore, $\mathcal{S} = \bigcup_{i \in \mathcal{Z}, j \in \mathcal{K}_i} \mathcal{S}_{ij}$ is used to denote the strategy space union for all the EVs in the studied region. For example, there are 6 zones illustrated in Figure 4.1, and we suppose there are chargers in all zones. If a group of EVs' daily travel routine

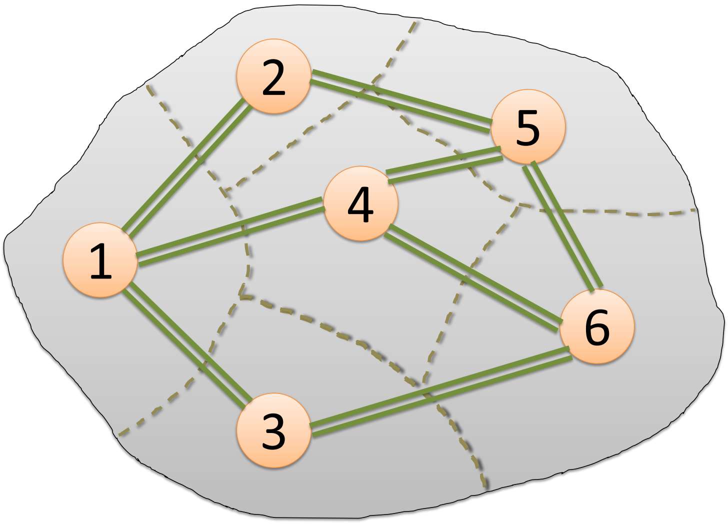


FIGURE 4.1: Zone division illustration

includes zones 1, 4 and 6, their strategy space includes these 8 pure strategies:

$$\mathrm{s}_1 = \{1\}, \quad \mathrm{s}_2 = \{4\}, \quad \mathrm{s}_3 = \{6\}, \quad \mathrm{s}_4 = \{2, (1,2), (2,1)\},$$
$$\mathrm{s}_5 = \{3, (1,3), (3,1)\}, \quad \mathrm{s}_6 = \{5, (4,5), (5,4)\},$$
$$\mathrm{s}_7 = \{5, (6,5), (5,6)\}, \quad \mathrm{s}_8 = \{3, (6,3), (3,6)\}.$$

When the charging zone is in their routine, there is no links in the strategy (e.g., s_1, s_2 and s_3). Otherwise (e.g., s_4 to s_8), there are two additional links specifying a round trip to the charging zone.

EVs choose charging strategy according to charging cost, including the charging fee, travel cost and queuing cost. In reverse, their strategies also influence congestion level and queueing time, i.e., travel cost and queueing cost. We denote the strategy distribution of EVs of group j in the zone i as $\mathbf{p}_{ij}$, with each $p_{ij}(\mathbf{s})$ denoting the proportion of γ_{ij} EVs using pure strategy s in the group of EVs' strategy space $\mathcal{S}_{ij}$.

Given a strategy profile $\mathbf{P} = \{\mathbf{p}_{ij}\}$, the number of EVs in each charging station $z \in \mathcal{Z}$ and the number of EVs on each link $e \in \mathcal{E}$ can then be seen as functions of the

$\mathbf{P}$ as:

$$f_z(\mathbf{P}) = \sum_{\mathbf{s}\in\mathcal{S}:z\in\mathbf{s}} \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{K}_i:\mathbf{s}\in\mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \tag{4.6}$$

$$f_e(\mathbf{P}) = \sum_{\mathbf{s}\in\mathcal{S}:e\in\mathbf{s}} \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{K}_i:\mathbf{s}\in\mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \tag{4.7}$$

Similarly, travel cost t for link e and zone z, and queuing cost q for zone z are respectively defined as:

$$t_e(\mathbf{P}) = \frac{d_e}{C_e}\left(f_e^0 + \sum_{\mathbf{s}\in\mathcal{S}:e\in\mathbf{s}} \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{K}_i:\mathbf{s}\in\mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \right), \tag{4.8}$$

$$t_z(\mathbf{P}) = \frac{d_z}{C_z}\left(f_z^0 + \zeta \sum_{\mathbf{s}\in\mathcal{S}:z\in\mathbf{s}} \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{K}_i:\mathbf{s}\in\mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}) \right), \tag{4.9}$$

$$q_z(\mathbf{P}) = \frac{1}{\tau_z} \sum_{\mathbf{s}\in\mathcal{S}:z\in\mathbf{s}} \sum_{i\in\mathcal{Z}} \sum_{j\in\mathcal{K}_i:\mathbf{s}\in\mathcal{S}_{ij}} \gamma_{ij} p_{ij}(\mathbf{s}). \tag{4.10}$$

Apart from the above costs, EVs also consider their charging fees, denoted by x_z, which vary at different zones. The weights ω_1, ω_2, and ω_3 are assigned to the three types of costs respectively. Thus, given the electricity price x_z in zone z, the charging cost of an EV using charging strategy s under strategy profile $\mathbf{P}$ is:

$$C_{ij}(\mathbf{P}, \mathbf{s}) = \sum_{z\in\mathbf{s}} \left(\omega_1 q_z(\mathbf{P}) + \omega_2 t_z(\mathbf{P}) + \omega_3 x_z \right) + \sum_{e\in\mathbf{s}} \omega_2 t_e(\mathbf{P}) \tag{4.11}$$

for all zone $i \in \mathcal{Z}$, group $j \in \mathcal{K}_i$ and pure strategy $\mathbf{s} \in \mathcal{S}_{ij}$.

4.3.2 Equilibrium

We adopt Nash equilibrium pin *non-atomic* congestion game as our solution concept. A non-atomic congestion game is one that is played by an uncountably large number of players (which is exactly the case in our problem, with around $30,000$ EVs in the system) so that each agent's effect on the congestion level is negligibly small. It is widely used to model congestion scenario with a large number of agents, which is exactly our case. In an equilibrium state, no EV can decrease her charging cost by unilaterally changing her charging strategy. Specifically, for each EV group j in zone i, the charging cost of all pure strategies that are used with non-zero probability are the same and the minimal, i.e.,

$$C_{ij}(\mathbf{P}, \mathbf{s}) \leq C_{ij}(\mathbf{P}, \mathbf{s}') \quad \forall \mathbf{s} \in \mathcal{S}_{ij},\ p_{ij}(\mathbf{s}) > 0 \tag{4.12}$$

In this case, no EV has incentive to unilaterally change her charging strategy.

4.3.3 Pricing Problem for EV Charging Station Management

As we have mentioned before, our goal is to minimize the social cost, denoted as SC. Specifically, we consider the extra social cost incurred by EVs' charging behavior, which is measured with the congestion experienced by all EVs in charging stations and extra congestion caused by EVs' charging behavior for *all* vehicles in the road network, i.e.,

$$SC = \nu_1 \sum_{z \in \mathcal{Z}} f_z(\mathbf{P}) q_z(\mathbf{P}) + \tag{4.13}$$

$$\nu_2 \sum_{z \in \mathcal{Z}} \left((f_z^0 + f_z(\mathbf{P})) t_z(\mathbf{P}) - f_z^0 t_z(\mathbf{0}) \right) + \tag{4.14}$$

$$\nu_2 \sum_{e \in \mathcal{E}} \left((f_e^0 + f_e(\mathbf{P})) t_e(\mathbf{P}) - f_e^0 t_e(\mathbf{0}) \right), \tag{4.15}$$

where the first component represents queuing cost for EVs in all zones, weighted with ν_1; and the second and the third components respectively represent additional travel cost in each zone and on each link for all vehicles, weighted with ν_2. It suffices for us to formulate the pricing problem for EV charging station management, which turns out to be a non-convex optimization problem **PCS** as follows.

$$\textbf{PCS}: \quad \min_{\mathbf{x},\mathbf{P}} \quad SC \tag{4.16}$$

$$\text{s.t.} \quad p_{ij}(\mathbf{s})C_{ij}(\mathbf{P},\mathbf{s}) \leq p_{ij}(\mathbf{s})C_{ij}(\mathbf{P},\mathbf{s}'), \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i, \forall \mathbf{s}, \mathbf{s}' \in \mathcal{S}_{ij} \tag{4.17}$$

$$\sum_{z \in \mathcal{Z}} f_z(\mathbf{P})x_z \geq B \tag{4.18}$$

$$x_z \in \mathcal{X}, \qquad \forall z \in \mathcal{Z} \tag{4.19}$$

$$p_{ij}(\mathbf{s}) \geq 0, \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i, \forall \mathbf{s} \in \mathcal{S}_{ij} \tag{4.20}$$

$$\sum_{\mathbf{s} \in \mathcal{S}_{ij}} p_{ij}(\mathbf{s}) = 1, \quad \forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i \tag{4.21}$$

Note that Equation (4.17) functions as the equilibrium criteria in Equation (4.12), i.e., when $p_{ij}(\mathbf{s}) = 0$, it holds unconditionally, and when $p_{ij}(\mathbf{s}) > 0$, it is equivalent to $C_{ij}(\mathbf{P},\mathbf{s}) \leq C_{ij}(\mathbf{P},\mathbf{s}')$; Equation (4.18) is a budget constraint requiring that the income of all charging station can at least cover their management and operation expenses; we suppose the charging rate in each zone is selected from a price set $\mathcal{X}$; and the last two constraints are to bound the p variables.

4.4 Computing Optimal Price

In this section, we present our algorithm for problem **PCS**. Problem **PCS** is a non-convex quadratic optimization problem, with its objective function SC being quadratic, and the first constraint (i.e., Equation (4.17)) being non-convex. Besides, the scale of problem **PCS** is very large because EVs have many travel patterns and, moreover, each pattern may contain many zones, which amounts to a large strategy space and a similarly large set of variables for problem **PCS**. Therefore, problem **PCS** is hard to solve (particularly hard to scale up). To resolve the problem, we first rewrite constraint (4.17), and reformulate **PCS** to the following *binary* programming **PCS**-$binary$ with an additional set of binary variables $\mathbf{y} = \langle y_{ij}(\mathbf{s})\rangle$.

PCS-$binary$:

$$\min_{\mathbf{x},\mathbf{P},\mathbf{y}} \quad SC \tag{4.22}$$

$$\text{s.t.} \quad y_{ij}(\mathbf{s})C_{ij}(\mathbf{P},\mathbf{s}) \le y_{ij}(\mathbf{s})C_{ij}(\mathbf{P},\mathbf{s}'),$$

$$\forall i \in \mathcal{Z}, \forall j \in \mathcal{K}_i, \forall \mathbf{s}, \mathbf{s}' \in \mathcal{S}_{ij} \tag{4.23}$$

$$p_{ij}(\mathbf{s}) \le y_{ij}(\mathbf{s}),\ \forall i \in \mathcal{Z}, j \in \mathcal{K}_i, \mathbf{s} \in \mathcal{S}_{ij} \tag{4.24}$$

$$y_{ij}(\mathbf{s}) \in \{0,1\},\ \forall i \in \mathcal{Z}, j \in \mathcal{K}_i, \mathbf{s} \in \mathcal{S}_{ij} \tag{4.25}$$

Eqs. (4.18) – (4.21)

Equations (4.23)–(4.24) are modified from Equation (4.17) with the auxiliary variable $\mathbf{y}$. As we can see, $y_{ij}(\mathbf{s})$ is an indicator for whether $\mathbf{s}$ can be used with *non-zero* probability in the solution, i.e., when $y_{ij}(\mathbf{s}) = 0$, we have (by Equation (4.24)) $0 \le p_{ij}(\mathbf{s}) \le 0 \Rightarrow p_{ij}(\mathbf{s}) = 0$, and when $y_{ij}(\mathbf{s}) = 1$, we have $0 \le p_{ij}(\mathbf{s}) \le 1$. Therefore, Equations (4.23)–(4.24) are equivalent to (4.17), and **PCS**-$binary$ is equivalent to **PCS**. To solve **PCS**-$binary$, a brute-force way is to exhaustively try all the 0/1 combinations in the feasible space $\{0,1\}^{|\mathbf{y}|}$ for $\mathbf{y}$. When $\mathbf{y}$ is fixed, **PCS**-$binary$ becomes a quadratic programming with linear constraints, which is relatively easy to solve. We then propose a Strategy Space Generation Algorithm (*SSGA*) to speed up the brute-force search, which is sketched with Algorithm 3.

SSGA starts with an initialized vector $\mathbf{y}^*$ (Line 1) and, repeatedly, solves **PCS**-$binary$ with $\mathbf{y}^*$, and updates $\mathbf{y}^*$ with two key procedures Rule A (Lines 6–8) and Rule B (Lines 11–15), until no update is made on $\mathbf{y}^*$ in some iteration (Line 16). Specifically,

- Rule A disables strategies $\mathbf{s}$ that are chosen with very small probability (i.e., $p_{ij}(\mathbf{s})<\delta$ with $0<\delta\ll1$) by setting $y_{ij}(\mathbf{s}) = 0$, so that they will not be used in the next iteration. The intuition behind Rule A is that when $p_{ij}(\mathbf{s})$ is very close to

Algorithm 3: SSGA

1 $\mathbf{y}^* \leftarrow$ Initialize as a binary vector;
2 $\mathbf{x}^* \leftarrow$ *Null*;
3 **repeat**
4     $\langle \mathbf{x}^*, \mathbf{P}^* \rangle \leftarrow$ Fix $\mathbf{y}$ to $\mathbf{y}^*$ and solve **PCS**-$binary$;
5     $\mathbf{y}' \leftarrow \mathbf{y}^*$;
    /* ---------------------- Rule A ---------------------- */
6   **for** *each* $i \in \mathcal{Z}$, $j \in \mathcal{K}_i$, $\mathbf{s} \in \mathcal{S}_{ij}$ **do**
7   **if** $p^*_{ij}(\mathbf{s}) < \delta$ **then**
8             $y^*_{ij}(\mathbf{s}) \leftarrow 0$;
9   **if** $\mathbf{y}^* \neq \mathbf{y}'$ **then**
10   **goto** Line 4;
    /* ---------------------- Rule B ---------------------- */
11   **for** *each* $i \in \mathcal{Z}$, $j \in \mathcal{K}_i$ **do**
12         $C^{min}_{ij} \leftarrow \arg\min_{\mathbf{s}:\, \mathbf{s} \in \mathcal{S}_{ij} \wedge p'_{ij}(\mathbf{s}) > 0} C_{ij}(\mathbf{P}^*, \mathbf{s})$;
13   **for** *each* $\mathbf{s} \in \mathcal{S}_{ij}$ **do**
14   **if** $y'_{ij}(\mathbf{s}) = 0$ *and* $C_{ij}(\mathbf{P}^*, \mathbf{s}) \leq C^{min}_{ij}$ **then**
15                 $y^*_{ij}(\mathbf{s}) \leftarrow 1$;
16 **until** $\mathbf{y}^* = \mathbf{y}'$;
17 **return** $\mathbf{x}^*$;

0, setting it to 0 will not cause much change to the cost of the EVs, but can significantly expand the feasible space as the associate constraint in Equation (4.23) is relaxed.

- Rule B checks if there are any unused strategies that could potentially lower EVs' cost, and enables them by setting $y_{ij}(\mathbf{s}) = 1$ when they are found. Intuitively, these newly enabled strategies are EVs' better responses to the current strategy profile.

Finally, Proposition 4.1 shows that *SSGA* always converges to a Nash equilibrium. The price $\mathbf{x}^*$ it returns is thus the optimal price under the equilibrium.

Proposition 4.1. *The algorithm SSGA always converges to an equilibrium charging strategy profile.*

Proof. Recalling that we use *Rule B* to validate whether or not we need to keep on the iteration of upgrading the set of used pure strategies and *Rule B* is exactly using the equilibrium criteria as stated in Equation (4.12), we can always ensure the strategy distribution $\mathbf{P}$ in the solution is an equilibrium. □

4.5 Experimental Results

In this section, experimental results are provided to verify the optimality and scalability of the proposed approach *SSGA*, and to present the improvement of traffic system performance provided by our approach. All computations are performed on a 64-bit machine with 16 GB RAM and a quad-core Intel i7-4770 3.4 GHz processor. All standard optimization problems, such as Line 4 of Algorithm 3, are solved with KNITRO 9.0.0.

Data of Singapore. We divide Singapore into 23 zones as Figure 1.1 according to the official planning-area information [136] and other geographic information. According to statistic data from the Department of Statistics, Ministry of Trade and Industry, Singapore [137], 23% of the residents usually drive to work. Besides, there are more than $972,000$ vehicles in the year 2014, among which more than $600,000$ are cars or station-wagons. We suppose 5% of the $600,000$ vehicles are EVs that charge in the charging game, then the total number of $30,000$ EVs are assigned to different zones according to the residential population distribution. The traffic flow and road capacity for each zone and link are estimated according to Google real-time traffic map. The price set is set as $\mathcal{X} = \{1, 1.5, 2, 2.5, 3\}$, according to the charging fee of charging stations in the U.S. [138]. The discount factor in computing the traffic congestion inside a zone is set as $\zeta = 0.5$. The weights for different parts in social cost and EVs' charging cost are set as: $\nu_1 = 0.8$, $\nu_2 = 0.2$; $\omega_1 = 0.1$, $\omega_2 = 0.3$ and $\omega_3 = 0.6$. The reason for this setting is to make sure that charging fee, travel cost and queuing cost are comparable. Except in Section 4.5.1, where we use a set of mock data of the traffic network and EV population to test the solution quality and scalability of *SSGA*, all other experiments are based on the data of Singapore.

Initializing the Binary Indicators for *SSGA*. The initial value of the indicators in Line 1 of the algorithm *SSGA* significantly influences the accuracy and speed of the approach. We apply the following method for initializing starting indicators for *SSGA*. We first compute the estimate the charging cost of each pure strategy assuming that there is only one EV that charges in the charging zone of that pure strategy and ignoring the charging fee. Formally, The estimate charging cost $\widetilde{C}_{ij}(\mathrm{s})$ for EVs of each pattern j in each zone i is computed as

$$\widetilde{C}_{ij}(\mathrm{s}) = \sum_{z\in \mathrm{s}} \left(\omega_1 \frac{1}{\tau_z} + \omega_2 \frac{d_z}{C_z}(f_z^0 + \zeta) \right) + \sum_{e\in \mathrm{s}} \omega_2 \frac{d_e}{C_e}(f_e^0 + 1). \tag{4.26}$$

Then we select the strategies whose estimated charging costs are no more than twice of the minimum of them and set their indicators as 1. That is to say, we set the indicator $y_{ij}(\mathrm{s}) = 1$ for all the $\mathrm{s} \in \mathcal{S}_{ij}$ with $\widetilde{C}_{ij}(\mathrm{s}) \leq 2\min_{\mathrm{s}'\in\mathcal{S}_{ij}} \widetilde{C}_{ij}(\mathrm{s}')$.

Virtual Charging Station Placement for Experiments on Data of Singapore. Since the charging station network in Singapore is not settled yet, we use virtual placements of EV charging stations for our experiments. For the total number of $30,000$ EVs in the charging game, we assign a total number of $2,000$ chargers to the charging stations in the region. Three kinds of placement are used according to the following rules, respectively.

A1 Placement according to population distribution. Namely, the chargers are distributed proportionally according to the population in each zone, i.e., $\tau_z \propto \gamma_z, \forall z \in \mathcal{Z}$.

B1 Placement according to current gas station distribution. This rule refers to the current gas station distribution in Singapore and assigns chargers proportionally according to the number of gas stations in each zone. Formally, assume that there are τ_z^{Gas} gas stations in zone z, the number of chargers in this zone $\tau_i \propto \tau_z^{Gas}$.

C1 Placement considering traffic congestion. The number of chargers in a zone is set proportional to the inverse of the normal congestion (regardless of the charging EVs) inside the zone, i.e., $\tau_z \propto \frac{C_z}{f_z^0}, \forall z \in \mathcal{Z}$.

4.5.1 Solution Quality and Scalability of SSGA

For experiments in this part, we generate a set of mock data, because we need problems of different scales to verify the optimality and scalability of *SSGA* through comparing with **PCS**.

Mock Data. We generate mock data using a Java program. First, the number of zones n is specified and the budget is set as $100n$. The traffic network is randomly generated by building a two-way link between any pair of zones with probability $4.5/n$. After the construction of the travel network, we randomly set the number of travel patterns in each zone as one of the elements in the set $\{1, 2, 3\}$ and we randomize the number of EVs of each pattern between 50 and 100. The traffic capacity and external flow (i.e., f_z^0 and f_e^0) in zones and links are randomized as integers in $[140, 160]$ and $[100, 200]$, respectively. For the charging stations' location and size, we use two methods to set them up. In the following are the two **charging station placement plans on mock data**.

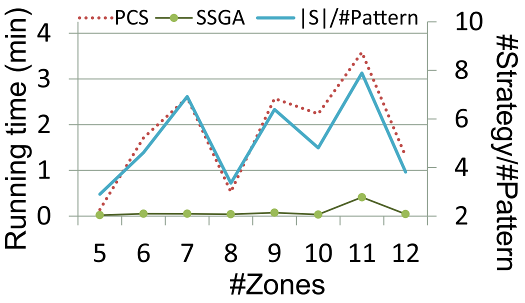
A2 We randomly choose some of the zones and assign 10 chargers to each of them. The expectation of the number of zones with chargers is $n/2$.

B2 We first calculate an index value θ_z for each zone $z \in \mathcal{Z}$ as following

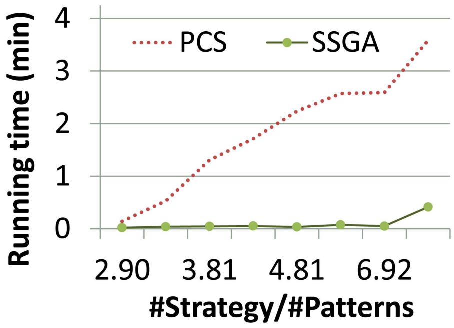
$$\theta_z = \sum_{i \in \mathcal{Z}} \sum_{j \in \mathcal{K}_i : z \in \mathcal{P}_{ij}} \gamma_{ij}.$$

This index value reflects how many EVs visit zone z frequently. Then we assign a number $5n$ of chargers to the zones proportionally, i.e., $\tau_z \propto \theta_z, \forall z \in \mathcal{Z}$.

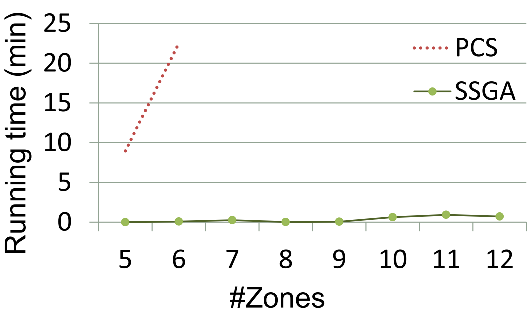
To test the optimality and scalability of *SSGA*, we generate different problems with n ranging from 5 to 12. Since the travel network, travel patterns and charging stations are randomly generated, the size of strategy space (i.e., $|\mathcal{S}|$), as well as the average size of strategy space for each travel pattern of EVs (i.e., $|\mathcal{S}|/|\mathcal{P}|$), are also randomized and does not have to increase with n (refer to "$|S|$/#Pattern" curve in Figure 4.2a for the variation trend). We then solve **PCS** and *SSGA* based on the above described mock data and the corresponding charging station placement **A2** and **B2**.



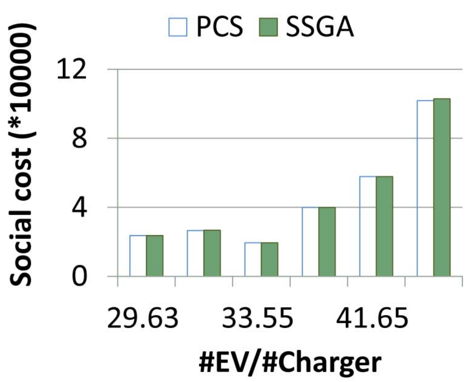
(A) Running time (**A2**)



(B) Running time trend (**A2**)



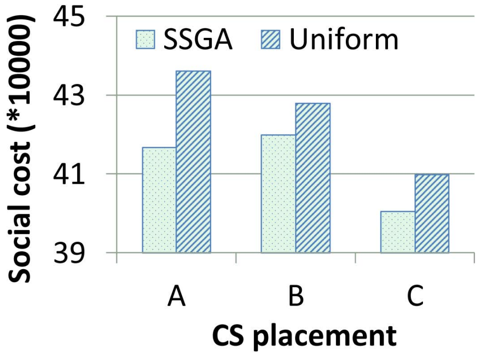
(C) Running time (**B2**)



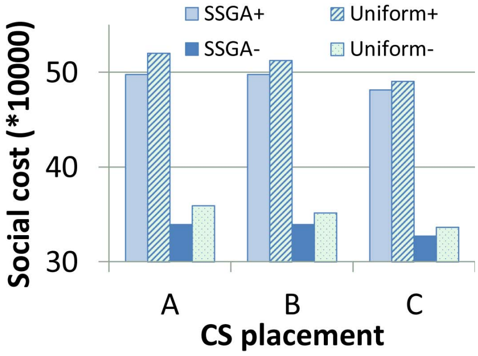
(D) Social cost (**A2**)

FIGURE 4.2: Optimality and scalability of *SSGA* comparing with **PCS**: (a) illustrates the running time of **PCS** and *SSGA* on the primary y axis, while on the secondary y axis showing the total number of pure strategies divided by the total number of EV patterns (i.e., the average size of strategy space for each pattern of EVs) corresponding to the charging station placement **A2**; (b) shows the relationship between the running time and the average size of strategy space for each pattern of EVs; (c) depicts running time results corresponding to the charging station placement **B2**; and (d) depicts the social cost of both **PCS** and *SSGA* for placement **A2**.

Experimental results are shown in Figure 4.2. When we increase the number of zones, the size of the strategy space also increases accordingly. In Figure 4.2a and Figure 4.2c, the running time of both approaches under different charging station placement plans is respectively described. It is shown that the running time of **PCS** does not monotonously increase with the number of zones but corresponds to the "$|S|$/#Pattern" curve. This is because the latter decides the complexity of the problem. For better visualization, we depict the relationship between running time and the average strategy space $|\mathcal{S}|/|\mathcal{P}|$ in Figure 4.2b, from which we can see that the running time increases



(A) Social cost



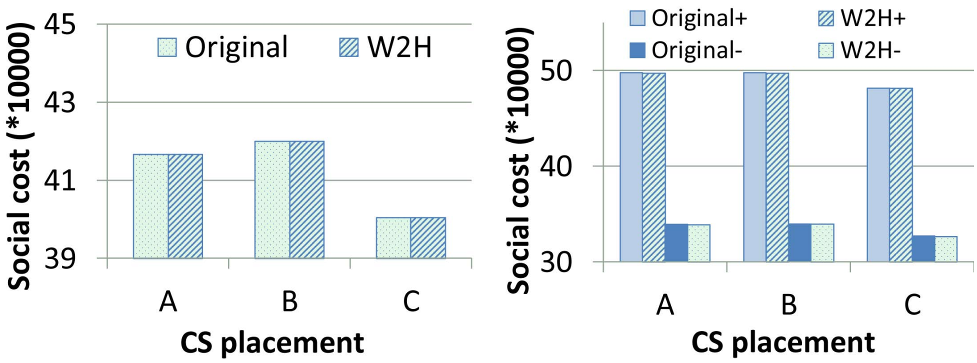
(B) Social cost w.r.t. # of EVs

FIGURE 4.3: Comparing *SSGA* with uniform pricing under different charging station placement plans **A1**, **B1** and **C1**: (a) the number of EVs is estimated as we stated before; and (b) we increase and decrease the number of EVs by 10%.

accordingly when $|\mathcal{S}|/|\mathcal{P}|$ increases. In Figure 4.2c, as we are using the charging station plan **B2**, the number of zones with chargers increases, so does the problem scale. In this case, **PCS** cannot handle the problem even for a small graph. Obviously, our approach *SSGA* drastically decreases the running time. As we can see from Figure 4.2d, *SSGA* always results in very close optimal social cost to **PCS**. Note that the social cost increases with the number of EVs divided by the number of chargers, i.e., the average number of EVs that a charger needs to serve.

4.5.2 Advantages over Uniform Pricing

We apply our approach to the data of Singapore and compare our pricing policy with the benchmark - uniform pricing (i.e., set charging rate in all the charging stations as the same), which represents no utilization of pricing measure for improving traffic system performance. We then conduct experiments according to different charging station placement plans **A1** to **C1** for both *SSGA* and uniform pricing. In Figure 4.3a, the number of EVs in each zone is set as we stated before. We can see that *SSGA* largely decreases the social cost, especially when the social cost is higher. We then increase/decrease the number of EVs in different extents to see what is the difference between the two methods when the EV density in the region is different. In Figure 4.3b, the legends “*SSGA*+” and “Uniform+” refer to the results when the number of EVs is increased by



(A) Social cost (B) Social cost w.r.t. # of EVs

FIGURE 4.4: Social cost of *SSGA* on original data and with two-step hop charging strategies: (a) the number of EVs is estimated as we stated before; and (b) the number of EVs in each travel pattern is increased/decreased by 10%.

10%; similarly, the legends "*SSGA*-" and "Uniform-" refer to the results when the number of EVs is decreased by 10%. We find that when the number of EVs increases, the advantage of distinct pricing computed by *SSGA* also increases.

4.5.3 Two-step Hop Charging Strategies

In this part, we release the assumption that EVs only charge in her routine zones or adjacent ones of them to see what will happen to the optimal social cost and the equilibrium strategy profile. We choose zone 1 for the test by adding up all the 2-step hop charging strategies s' for the each patterns j EVs in zone 1:

$$\mathrm{s}' = \{z'', (z, z'), (z', z''), (z'', z'), (z', z)\}, \tag{4.27}$$

where z is a zone in the travel pattern $\mathcal{P}_{1j}$, but z' and z'' are not. We then conduct experiments on original data and data with varied number of EVs (increas/decreas by 10% as we used in Section 4.5.2). We find that although the two-step hop strategies are added to the EVs' strategy space, they are never used and the optimal social cost never changes, which is shown in Figure 4.4. Furthermore, we find that the charging cost of those two-step hop strategies is much larger than those employed strategies. We conclude that it is reasonable to ignore these strategies.

4.5.4 Adaption to Population Change

The population in cities changes in amount as well as in distribution along with the city development. A concrete example is what we show in Section 4.1 about Singapore. Once the charging stations are settled in the city, it is costly to modify their layout although that the traffic system performance will decrease along with city development, which directly leads to citizens' travel pattern change. Thus we propose to use adaptive dynamic pricing to accommodate changes in population density and travel patterns, thus to mitigate the traffic congestion and decrease the social cost.

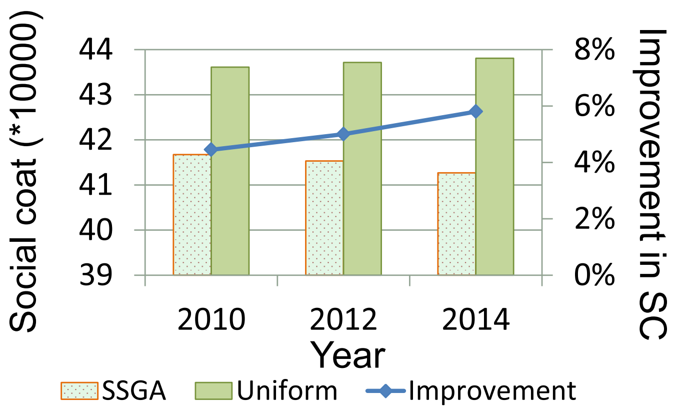
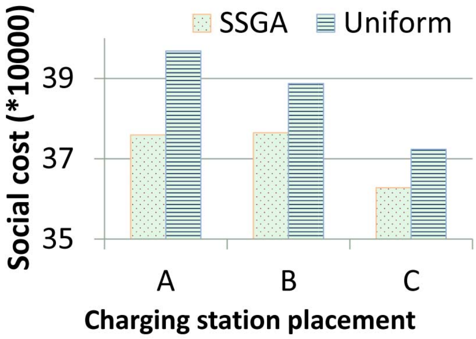
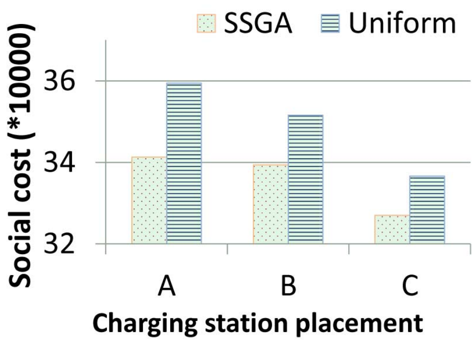


FIGURE 4.5: The optimal social cost of *SSGA* and uniform pricing, as well as the system efficiency improvement along with time.

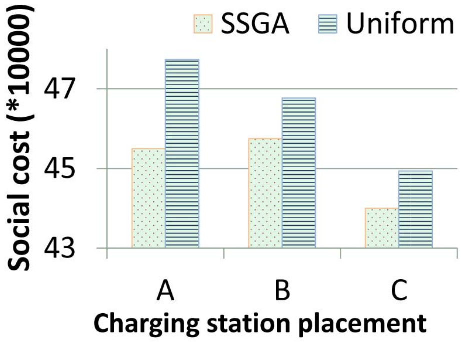
We first arrange the charging stations according to plan **A2** for Singapore in the year 2010. Based on that charging station placement, we then conduct experiments for population distribution in the year 2012 and 2014, respectively. The results in the social cost of both *SSGA* and uniform pricing are depicted in Figure 4.5, where the x axis denotes the year, and the primary and secondary y axes denote social cost and the percentage of improvement in social cost, respectively. As we can see, *SSGA* is quite adaptive to the population density change and it always decreases the social cost by a considerable amount. Furthermore, the "Improvement" curve shows that the decrease in social cost (i.e., improvement in system performance) increases with time. It turns out that when the system degenerates, *SSGA* performs even better.



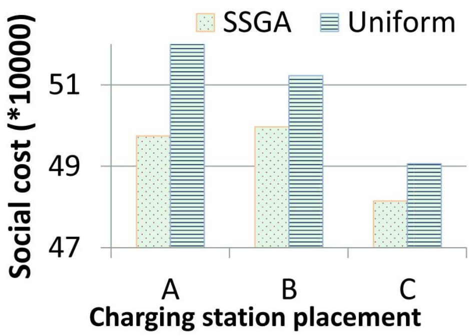
(A) # of EVs decrease by 5%



(B) # of EVs decrease by 10%



(C) # of EVs increase by 5%



(D) # of EVs increase by 10%

FIGURE 4.6: Robustness test of *SSGA* comparing with uniform pricing regarding to deviation in estimation of the number of EVs: (a) the actual number of EVs of each pattern is 5% less than the estimation; (b) the actual number of EVs of each pattern is 10% less than the estimation; (c) the actual number of EVs of each pattern is 5% more than the estimation; and (d) the actual number of EVs of each pattern is 10% more than the estimation.

4.5.5 Sensitivity and Robustness

In the above sections, the number of EVs in each zone of each pattern is accurately estimated. Considering that there might be some deviation between estimation and true values, we test the sensitivity of our approach regarding the number of EVs and consider the case that the estimated number of EVs is not accurate. We first compute the optimal price $\mathbf{x}$ according to our estimation. Then we compute the social cost with fixed $\mathbf{x}$ and EV number deviation by letting $\gamma_i' = \gamma_i(1 \pm \varepsilon)$ with $\epsilon = 5\%$ or 10%.

As we can see in Figure 4.6, *SSGA* always achieves better performance (i.e., results in lower social cost) than uniform pricing even when the estimation is not precise. Thus our pricing policy is robust regarding the uncertainty of estimation of the number of

EVs. Besides, according to the social cost of uniform pricing and the "Improvement" curve, we can see that when the social cost is higher, *SSGA* actually outperforms uniform pricing more.

4.6 Summary

In this chapter, we take a game-theoretic perspective to study the EV charging station pricing problem motivated by the practical need of EV promotion in Singapore. Our first contribution in this work is a novel pricing model that comprehensively incorporates EV users' self-interested charging behavior and their various traffic patterns, traffic congestion contributed by EVs and other non-EV vehicles in the road network, as well as the financial concern for a sustainable operation of the charging network. The second contribution is the algorithm, *SSGA*, to solve the mixed integer non-convex optimal pricing problem, which features two key rules that guarantee efficient converging to equilibrium solution and drastically improves the running time performance. The final contribution is our extensive experiments and results which demonstrate our approach in several aspects, including solution quality, scalability, and robustness. Moreover, we compare our approach with uniform pricing and demonstrate how and to what extent *SSGA* can help with improving the traffic system efficiency and decreasing social cost caused by EV owners' charging behavior. Our approach can be applied in various modern cities like the motivating example Singapore to manage the charging stations in the future. We are actively approaching authorities of Singapore to look for such potential application.

There are two aspects from which we can further improve our research. First, we assume the charging price to be discrete since it is more common in our daily life. Nevertheless, along with the development of electronic payment, people can pay the charging fee more convenient. As a result, we can relax the assumption and select the optimal charging fee from a continuous interval. We might need to study how to do that, and analyze and compare its performance with the current approach. Then, we adopt non-atomic congestion solution concept as the solution concept of our charging game, and thus the players in the same zone are identically treated regarding their charging strategies. The players are assumed to be fully rational and able to adjust their strategies until the equilibrium reached. However, in real-world scenarios, people can hardly do so due to their limited computational ability and observation etc. Therefore, more specific study for the EV drivers' charging behavior is required. In the next chapter, we present the charging behavior study and show an application on charging station placement.

Chapter 5

Charging Behavior Analysis and Optimization for Electric Vehicle Charging Station Placement

Electric vehicles (EVs) are attracting growing interest from the public in recent years. A critical limitation for promoting EVs is the limited battery capacity, which brings mileage anxiety for drivers. Consequently, the EV charging stations, which can support EVs with fast and convenient charging (around 20 to 30 minutes, usually 12 times faster than charging with domestic electricity) is important for the successful promotion of EVs. With planned charging stations, EV drivers can select the most suitable one to use according to the features of the charging stations, including the charging fare and queuing condition. In this process, the charging behavior of EV drivers, in return, affects the performance of the charging stations, i.e., the queuing condition. Thus, it is important to study the interrelationship between EV drivers and charging stations.

While there have been a number of works [7–10] on Charging Station Placement Problems (CSPP), only a few of them [7, 10] consider the EV drivers' charging behavior. However, their behavior models are based on simplistic assumptions. First, existing charging behavior models are lack of comprehensive study of EV drivers' preference over different factors; second, they assume that EV drivers are fully rational. In this paper, through carefully analyzing the charging behavior of EV drivers, we propose a realistic 2-Level Nested QRE (quantal response equilibrium) charging behavior model, which is the first contribution of this work. Each EV driver is trying to minimize his/her charging cost while making the decision and competing with each other for using the charging stations. Our 2LNQRE charging behavior model is inspired by the QRE model [102] and level-k thinking model [110]. To the best of our knowledge, we are the first to study EV drivers' specific charging behavior.

Second, to verify the 2LNQRE charging behavior model and learn its parameters, we carefully design a set of charging behavior user studies to simulate the charging scenarios in the real world. We collect human players' charging choices and learn the charging behavior model.

The third contribution of this paper is that we formulate the charging station placement problem with the 2LNQRE charging behavior model and design an efficient algorithm for the complex optimization problem. We utilize the approximate derivative and design a gradient descent approach.

The last contribution is that we conduct extensive experimental evaluations to compare 2LNQRE charging behavior model with several benchmark models and our placement approach with two benchmarks. It is shown that the proposed charging behavior model well captures the bounded rationality of EV drivers and our approach for placement significantly outperforms the benchmarks by decreasing the EV drivers' queuing time by at least 5%.

5.1 Charging Behavior Model

In this section, we first briefly introduce the QRE model, then introduce the proposed 2LNQRE charging behavior model, and then we present the design of user studies and the method to learn the model with collected data.

5.1.1 QRE Model

During the charging process, EV drivers are competing with each other to use the charging stations and they are self-interested in minimizing their own cost. We employ the framework of the congestion game to model the interactions among EV drivers. We define a charging game with following components.

- ***Player*** The EV drivers are players of the charging game.
- ***Strategy*** A strategy is a charging station that a player could use. In the charging game, players have a strategy set, i.e., a set of accessible charging stations. Based on all players' selection, their strategies form a distribution over the strategy set.
- ***Cost*** Considering that charging in different charging stations bring the same utility for EV drivers, i.e., having the EV recharged, we only need to consider their cost in the charging process that may vary from one station to another.
- ***Equilibrium*** Players are self-interested, and they would like to use the best strategy to minimize their cost. An equilibrium is the state of the game where no player would deviate.

Considering the charging process of an EV driver, he/she needs to drive from home to the charging station, (probably) queue in the charging station for some time and pay the charging fare before starting charging the EV. Thus, we consider the following factors that may influence EV drivers' charging cost[1]:

- t – the travel time on the road;
- d – the travel distance from the start point to the charging station;
- f – the charging fare in the charging station;
- q – the queuing time in the charging station.

[1]We list the most common factors that influence EV drivers' charging behavior. While other factors may exist, we can extend our model accordingly.

Assume that there are a number of choices for the players in the charging game. For each choice i, we model the charging cost function c_i as a linear combination of the above factors. Note that the weight can be 0, which means the corresponding factor does not influence the EV drivers' choices.

$$c_i = w_t t_i + w_d d_i + w_f f_i + w_q q_i \tag{5.1}$$

With the charging cost function in Eq.(5.1), we can denote the selection distribution of players according to quantal response equilibrium (QRE) model with Eq. (5.2), where p_i is the probability of choosing choice i.

$$p_i = \frac{e^{-\lambda(w_t t_i + w_d d_i + w_f f_i + w_q q_i)}}{\sum e^{-\lambda(w_t t_j + w_d d_j + w_f f_j + w_q q_j)}} \tag{5.2}$$

Note that the queuing time q_i is actually decided by the number of EV drivers in the charging station, thus can be rewritten as $q_i(\mathbf{p})$. Furthermore, the charging cost function can be denoted as $c_i(\mathbf{p})$.

When $\lambda \to 0$, players tend to be irrational and choose one option randomly; when $\lambda \to \infty$, players tend to be rational and choose the option with the lowest cost. As far as we know, there is no previous work studying the parameters of the charging cost function. Besides, the weights $\mathbf{w}$ are multiplying the rationality parameter λ. When the weights are doubled, it is equal to λ is halved. Thus we can assume $\lambda = 1$ and focus on the unknown weights. Therefore, in the following, we would eliminate the parameter λ for simplicity.

5.1.2 2LNQRE Model

A strong assumption in above QRE model is that players can form QRE distribution $\mathbf{p}$ according to the queuing time $\mathbf{q}(\mathbf{p})$. In reality, players can hardly know $\mathbf{q}(\mathbf{p})$. In reality, players would observe the situation, and then make a choice accordingly; furthermore, some players would even think further to anticipate others' charging choices before making a decision. Thus we propose a 2-Level Nested QRE (2LNQRE) model to capture human players' charging behavior by combining these two levels of thinking modes.

For the level-1 players, we assume that they would form a QRE distribution according to their observation, i.e., the current queuing time that they know at the moment, which is not influenced by their competitors. We denote this queuing time as $\hat{\mathbf{q}}$. Then, level-1 players would form the following QRE distribution $\hat{\mathbf{p}}$.

$$\hat{p}_i = \frac{e^{-c_i(\hat{\mathbf{q}})}}{\sum e^{-c_j(\hat{\mathbf{q}})}} \tag{5.3}$$

Consequently, level-2 players would anticipate $\hat{\mathbf{p}}$ and perceive queuing time as $\tilde{\mathbf{q}}(\hat{\mathbf{p}})$. The QRE distribution formed by level-2 players is then presented with Eq.(5.4).

$$\tilde{p}_i = \frac{e^{-c_i(\hat{\mathbf{p}})}}{\sum e^{-c_j(\hat{\mathbf{p}})}} \tag{5.4}$$

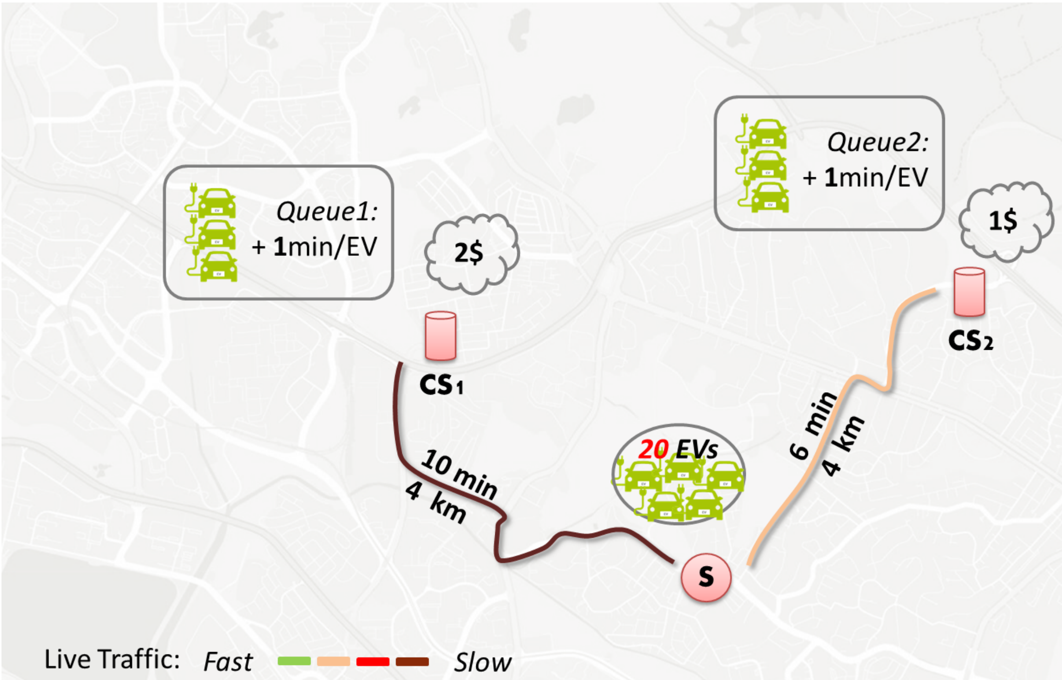
In reality, there would be a proportion of level-1 players and the rest are level-2 players. We assume that the proportion of level-1 players is γ. Then, the actual distribution of the players' charging choices is

$$p_i = \gamma \hat{p}_i + (1-\gamma)\tilde{p}_i \tag{5.5}$$

5.1.3 User Study Design

To learn the charging cost function and the level of rationality of EV drivers, we design a set of user studies to simulate the charging scenarios and collect data from human players.

We present charging scenarios for players with abstracted information as shown in Figure 5.1. The interface includes (1) a decolored map as background, (2) the start point marked with red circled S where EV drivers (players) stay, (3) the number of EV drivers at the start point that will go for charging at the same time, (4) the candidate charging stations marked with purple icons and named as $CS_i, i \in \{1, 2, ...\}$, (5) the charging fare in \$ at each charging station circled near the corresponding charging station, (6) the hint for queuing time[1], (7) the charging routes from the start point to each candidate charging station, along which the travel time and distance are denoted in min and km respectively, and (8) a table with text information below the map.



Your Charging Decision

(Note: the queuing time at each charging station increases 1min per extra EV driver!)

There are two choices for the 20 EV drivers at the start point:

	Travel time	Distance	Fare	Queuing time
CS1	10 min	4 km	2$	x min queue if x EVs choose it
CS2	6 min	4 km	1$	x min queue if x EVs choose it

FIGURE 5.1: User study interface example

The travel time and the distance from the start point to each of the charging stations are assumed to be stationary. To visualize the travel speed in the charging route, inspired by Google Maps, we use four colors to draw the travel routes (respectively representing travel speed around $20km/h$, $30km/h$, $40km/h$ and $50km/h$). All candidate charging stations are assumed to be located beside a shopping center, so there is no need for the players to consider which one is more convenient. The EV drivers at the same start point will charge at the same time, thus they would cause congestion in the charging

[1] The design of the game has been performed in several iterations with studies on human players to verify that they are aware of all the games' parameters.

stations. The EV drivers in a charging station with x EV drivers choosing it would wait for x mins on average before starting charging.

With all the information provided, a player is able to see the difference between different charging choices and then make his/her charging decision. For example, if a player at the start point S chooses the charging station CS_1, meanwhile there are 12 other EV drivers that also select CS_1, the player would travel 13 minutes, 9 kilometers, pay 1\$ and queue 13 minutes before charging.

5.1.3.1 One-Shot Charging Game

To study the charging behavior of EV drivers, we design two different charging scenarios $\{I_A, I_B\}$ respectively with (1) one start point and two candidate charging stations (I_A) and (2) one start point and three candidate charging stations (I_B). For each user study structure, we carefully design 6 scenarios with different parameters. The basic idea is to provide distinct scenarios to avoid over-fitting. The total 12 user studies are divided into two groups. Each invited human player is randomly directed to one of the two groups and makes a choice in each of the 6 different scenarios *for once*.

We put the user studies on a virtual machine build on Microsoft Azure platform. Players can access the user studies via a link[1]. They would first see the introduction with a video and also text explanation. The details of the user study is clearly presented to make sure players understand every part of it. After providing some general information, players would see a toy example on the next page, which is used to ensure that the participants have fully understood the user study. Only the players who can correctly answer the toy example question would be directed to the formal user studies. After that, players would see the interfaces of different charging scenarios and submit their selections.

[1]http://52.187.51.242/welcome

5.1.3.2 Repeated Charging Games

In the one-shot charging games, players have no idea about others' choices. Considering that people can have history knowledge from day to day in real-world scenarios, we furthermore conduct repeated charging games as follows. We selected one group of user studies to conduct repeated charging game. In this case, a number of players are required to make charging decisions simultaneously. In each round of a user study, firstly, all players submit their choices; then, the webpage would automatically refresh and present the result of this round, i.e., the resulted queuing time in each charging station. With this information, players would go to the next round for the same user study until the last round, after which they would see the next user study. In repeated charging games, players can get real-time feedback on their decisions and use this information for decision-making in the next round.

5.1.4 Parameter Estimation

We employ the Maximum Likelihood Estimation (MLE) to learn the parameters $\mathbf{w}$. For a charging scenario S with M records from the players, the logarithmic likelihood of $\mathbf{w}$ is

$$\log L(\mathbf{w}|S) = \sum\nolimits_{i=1}^{M} \log p_{cs(i)}(\mathbf{w}) \tag{5.6}$$

where $cs(i)$ is the charging station selected in the sample i. Assuming that there are K charging stations (strategies) for the players and the number of players that choose to use the k^{th} one is M_k, then we have $\sum_k M_k = M$ and

$$\log L(\mathbf{w}|S) = \sum\nolimits_{k=1}^{K} M_k \log p_k(\mathbf{w}) \tag{5.7}$$

By substitute p_k with Eq.(5.5), we have

$$\log L(\mathbf{w}|S) = \sum\nolimits_{k=1}^{K} M_k \log(\gamma\hat{p}_i + (1-\gamma)\tilde{p}_i) \tag{5.8}$$

In fact, we design a number of charging scenarios with different environment settings to study players charging behavior. Thus, we are maximizing the sum of the logarithmic likelihood when learning the parameters.

$$\log L(\mathbf{w}) = \sum\nolimits_S \log L(\mathbf{w}|S) \tag{5.9}$$

The function *fmincon* of Matlab is used for the maximization.

5.2 Charging Station Placement Problem

To model the Charging Station Placement Problem (CSPP), we divide the target area (e.g., a city) into n zones according to the geographic constraints and residential condition (see Figure 5.2 for an example on Singapore city). The set of zones are denoted as $\mathcal{N} = \{1, ..., n\}$, where each zone $i \in \mathcal{N}$ has a carefully selected candidate position to construct a charging station and the size of the charging station can be denoted as x_i. When $x_i = 0$, it means that zone i will not have a charging station; when $x_i > 0$, x_i represents the capacity of the charging station. The charging fare of the charging station in zone i is set as f_i, which is decided by the electricity cost of that zone. A budget B is used to constrain the financial support.

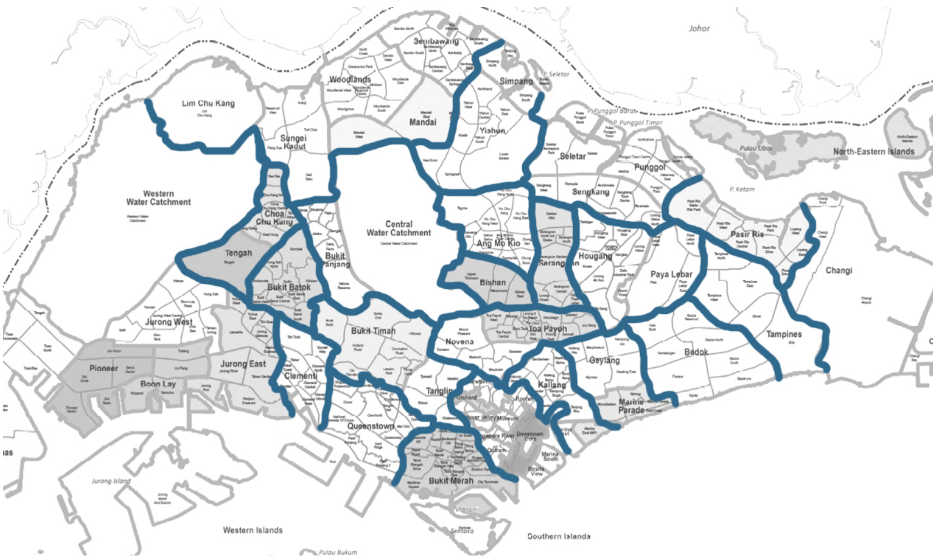


FIGURE 5.2: Zonal division of a target area

Meanwhile, in each zone i, there are a number of residents that own EVs, which can be denoted as E_i. A proportion $\tau \in [0, 1]$ of EV drivers are possible to go out for charging during the same time period. We assume that the EV drivers can charge in their residential zone and geographically adjacent zones (denoted with a set $\mathcal{A}_i$).[1] Then, the EV drivers from the same zone are treated as identical players with the same strategy set $\mathcal{A}_i$. Respectively, for a player in zone i that uses strategy j, the charging cost is denoted as c_{ij}. The strategy profile, i.e., charging selection distribution of players in zone i is denoted as $\mathbf{p}_i = \{p_{ij}\}$, where p_{ij} indicates the probability of using strategy j.

The queuing time q_j in charging station j is decided by the charging station size x_j and the number of EV drivers y_j that use it, which can be calculated with all players' strategy profile P.

$$y_j = \sum\nolimits_{i|j\in\mathcal{A}_i} E_i \tau p_{ij} \tag{5.10}$$

Assume that all chargers in charging stations have the same service ability, and take μ minutes to recharge one EV. The queuing time q_j for EV drivers in the charging station j is defined as following.

$$q_j = \frac{y_j \mu}{2x_j} = \frac{\mu \sum_{i|j\in\mathcal{A}_i} E_i \tau p_{ij}}{2x_j} \tag{5.11}$$

We consider the government as the investor to construct the charging stations, and the objective is to optimize the charging station performance, in other words, to minimize the social cost SC. Then, the CSPP is to minimize SC by strategically deciding the charging station placement $\mathbf{x} = \{x_i\}$ with respect to the EV drivers' charging behavior in the charging activities.

With the proposed 2LNQRE model, the CSPP can be formulated as follows.

$$\min_{\mathbf{x}} \quad SC = \sum\nolimits_{i\in\mathcal{N}} E_i \tau \sum\nolimits_{j\in\mathcal{A}_i} p_{ij} q_j(\mathbf{p}) \tag{5.12}$$

$$\textbf{s.t.} \quad \hat{p}_{ij} = \frac{e^{-(w_t t_{ij} + w_d d_{ij} + w_f f_j + w_q \hat{q}_j)}}{\sum e^{-(w_t t_{ik} + w_d d_{ik} + w_f f_k + w_q \hat{q}_k)}} \tag{5.13}$$

$$\tilde{p}_{ij} = \frac{e^{-(w_t t_{ij} + w_d d_{ij} + w_f f_j + w_q \tilde{q}_j(\hat{\mathbf{p}}))}}{\sum e^{-(w_t t_{ik} + w_d d_{ik} + w_f f_k + w_q \tilde{q}_k(\hat{\mathbf{p}}))}} \tag{5.14}$$

$$p_{ij} = \gamma \hat{p}_{ij} + (1-\gamma)\tilde{p}_{ij} \tag{5.15}$$

$$\sum\nolimits_{i\in\mathcal{N}} x_i \leq B, x_i \in \mathbb{N} \tag{5.16}$$

[1] The assumption can be extended to the case where EV drivers would charge on their way between home and working places. Our model can adapt to that by changing the set of choices for EV drivers from "the set of adjacent zones" to "the set of zones on their way between home and the working place".

The CSPP is an integer non-convex optimization problem, finding the global optimum is extremely hard. Therefore, we propose an algorithm named MAGD (Algorithm 4) with multiple start points, approximate derivatives and gradient descent method to compute an approximate solution. We first randomly generate a set of start points $\mathcal{S}$. In each iteration of a start point $\bar{\mathbf{x}}$, we first compute the corresponding social cost[1] $\bar{O}_{bj}$; for each step size from the maximum one N_S to 1, we iteratively update the $\bar{\mathbf{x}}$ to $\mathbf{x}$ w.r.t the approximate derivative $\bigtriangledown\bar{\mathbf{x}}$; then we compute O_{bj} with $\mathbf{x}$, compare it with the current smallest social cost O^*_{bj} and go to next iteration until $I_{iter} = N_I$ or there is no improvement for any $step$ in the current iteration. By increasing the number of start points and expanding the searching space, the probability of reaching the global optimal solution would be increased.

Algorithm 4: MAGD - Multi-start Approximate Gradient Descent Algorithm

Generate $\mathcal{S}$;
$O^*_{bj} \leftarrow \infty$;
for $\bar{\mathbf{x}} \in \mathcal{S}$ **do**
    Compute $\bar{O}_{bj}$ with $\bar{\mathbf{x}}$;
    for $I_{ter} = 1 : N_I$ **do**
        for $step = N_S : -1 : 1$ **do**
            $\bigtriangledown\bar{\mathbf{x}}_i \leftarrow \frac{-step \cdot y_i^2}{x_i(x_i+step)}$;
            $\mathbf{x} \leftarrow \bar{\mathbf{x}}$;
            for $i \in \mathcal{N}$ **do**
                $x_i \leftarrow \bar{x}_i + step$, if $\bigtriangledown x_i = \min_j \bigtriangledown x_j$;
                $x_i \leftarrow \bar{x}_i - step$, if $\bigtriangledown x_i = \max_j \bigtriangledown x_j$;
            Compute O_{bj} with $\mathbf{x}$;
            if $O_{bj} < O^*_{bj}$ **then**
                $\bar{\mathbf{x}}, \mathbf{x}^* \leftarrow \mathbf{x}$;
                $O^*_{bj} \leftarrow O_{bj}$;
                Goto next I_{ter};
            else
                Goto next $step$;;
return O^*_{bj} and $\mathbf{x}^*$;

[1] We employ KNITRO 9.0.0 and set *option* with "algorithm=5".

5.3 Experimental Results

In this section, we can see the experimental results from 50 records (25 records for each group of user studies) for the one-shot charging games. Besides, we invite 10 players to participate in the repeated charging games to play each of the 6 games from group A for 20 rounds. The queuing time is calculated w.r.t. the players' selection. In one-shot charging games, for a setting with M EVs from the start point, the actual queuing time in a candidate charging station is calculated and rounded as $q = [M \cdot N_i/N]$ min; for repeated charging games, $q = 2N_i$.

5.3.1 2LNQRE Model VS. QRE Model

For one-shot charging games, there are 12 plays (1 round for each of the 12 user studies). We take two measures to evaluate the prediction performance, i.e., mean squared error M_{SE} and Kullback-Leibler divergence D_{KL}, which is commonly used for measuring how one probability distribution diverges from another. Specifically, for two charging strategies $\mathbf{p} = \{p_i\}$ and $\mathbf{p}' = \{p'_i\}$, $D_{KL} = \sum_i (p_i \log \frac{p_i}{p'_i} + p'_i \log \frac{p'_i}{p_i})$. We test $\gamma = \{0.1, 0.2, ..., 0.9\}$ and find the optimal one is $\gamma = 0.8$. The learning results ($\mathbf{w}$ and learning errors) for QRE and 2LNQRE are presented in Table 5.1. As we can see, 2LNQRE model better captures the players' charging behavior with less error and more realistic weights, especially the weight of queuing time.

TABLE 5.1: Learning results comparison for one-shot charging games

	w_t	w_d	w_f	w_q
QRE	0.1347	0.0577	0.5545	0
2LNQRE	0.2479	0.0797	0.9969	0.1367

Errors	M_{SE}(mean/std)	D_{KL} (mean/std)
QRE	0.0062 / 0.0058	0.0621 / 0.0648
2LNQRE	0.0049 / 0.0051	0.0490 / 0.0519

In one-shot charging games, players have no information about the others' choices. In reality, players have experience from the past and might take that into consideration. Thus, we conduct repeated charging games for 120 plays (20 rounds for each of the 6 user studies). For repeated charging games, we assume that level-1 players

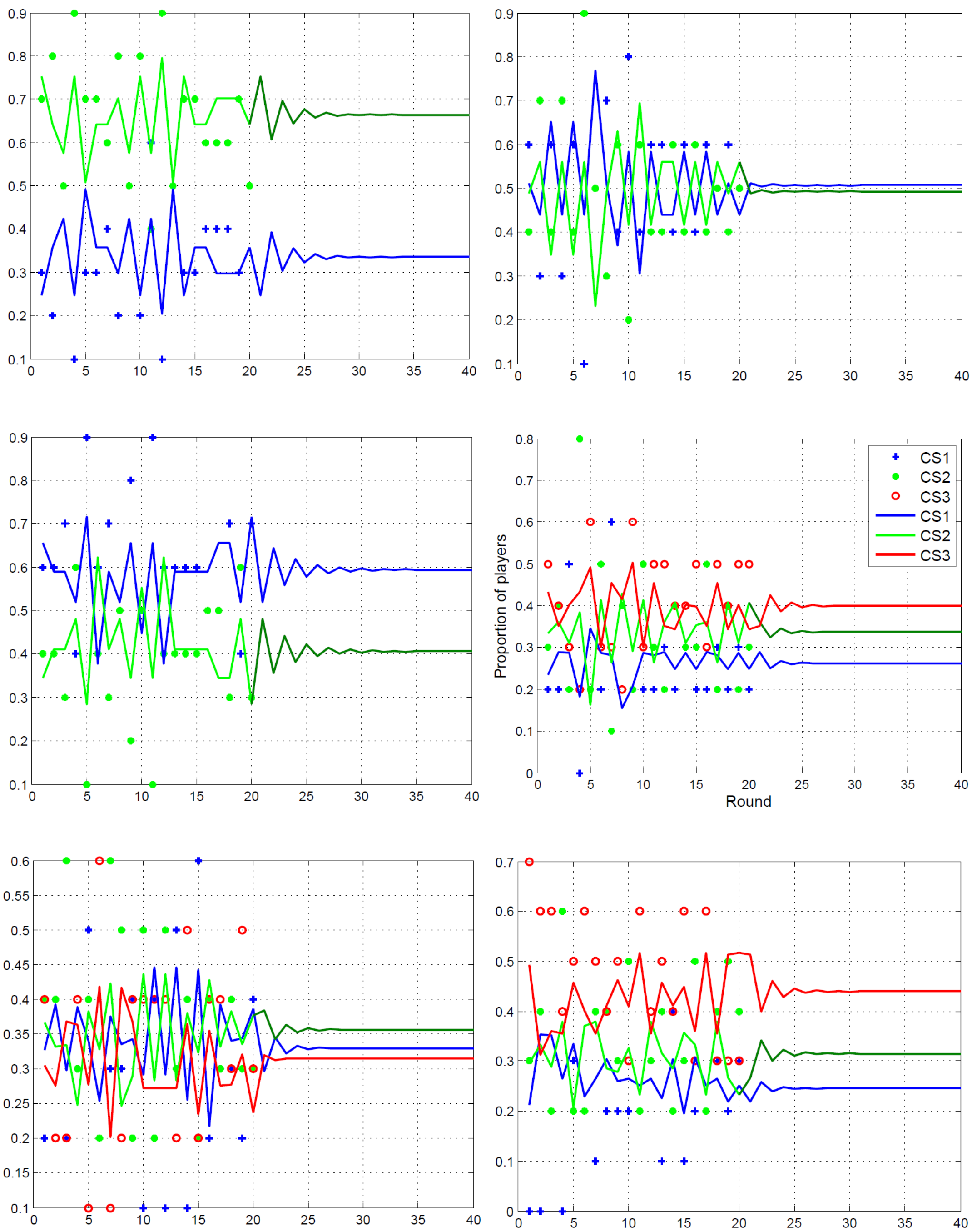


FIGURE 5.3: Players' selection distribution in 20 rounds of repeated games (scatter plots); prediction of 2LNQRE model for 40 rounds (curves)

perceive queuing time as $\hat{\mathbf{q}}$ (the weighted queuing time from previous m rounds). At the beginning of round r, the weighted queuing time of previous m rounds in each C-S i is $\hat{q}_i^r = \sum_{j=1}^{m} \alpha_j q_i^{r-j}$, where q_i^{r-j} is the actual queuing time at round $r-j$. We enumerate the combination of γ, $m \in \{1, 2, ..., 5\}$ and $\alpha_1 = \{0.5, 0.6, ..., 0.9\}$ (thus, $\alpha_k = \alpha_1(1 - \sum_{l=1}^{k-1} \alpha_l)$ except $\alpha_m = 1 - \sum_{l=1}^{m-1} \alpha_l$). Through comparing the learning errors, the best combination $m = 1, \gamma = 0.9$ (then α is eliminated) is selected. The learning results[1] for QRE and 2LNQRE model are presented in Table 5.2. Comparing with the results of one-shot charging games, we can see that our 2LNQRE model has greater advantage versus the QRE model in repeated ones, which has 10 times of number of plays.

TABLE 5.2: Learning results comparison for repeated charging games

	w_t	w_d	w_f	w_q	M_{SE}(mean/std)
QRE	0.0997	0.0028	0.2719	0	0.0577 / 0.0704
2LNQRE	0.1570	0.0080	0.4504	0.0749	0.0148 / 0.0190

Furthermore, we test whether and how the distribution of players' charging decision would converge. As we can see from Figure 5.3, the selection distribution would converge in several rounds. Table 5.3 presents the final convergence results of each of the 6 charging scenarios of the repeated charging settings.

TABLE 5.3: Convergence distribution of the 2LNQRE model

Game	CS1	CS2	CS3
1	0.3356	0.6644	—
2	0.5070	0.4930	—
3	0.5940	0.4060	—
4	0.2625	0.3375	0.4000
5	0.3293	0.3563	0.3144
6	0.2456	0.3149	0.4395

5.3.2 Comparison with Single-Level QRE Models

We compare the 2LNQRE model with two single-level QRE models: (1) Level-1 QRE model, which assumes that all players are at level-1 and form QRE distribution w.r.t.

[1]Note that D_{KL} is not computed since there are some CSs with no players selecting them, i.e., 0 probability.

their observation of queuing time; and (2) Level-2 QRE model, which assume that all players are at level-2.

We conduct learning process on repeated charging game results. Similarly, we traverse different combination of parameters m, α and find the optimal value for them, i.e., $m = 2, \alpha_1 = 0.5$. The learning results for these two models are shown in Table 5.4. Both of these models result in larger learning error compared with the 2LNQRE mode, which means level-1 and level-2 players coexist in reality.

TABLE 5.4: Results of Level-1 ("L1") and Level-2 ("L2") QRE models

	w_t	w_d	w_f	w_q	Error(mean/std)
L1	0.1301	0.0046	0.3620	0.0401	0.0639 / 0.0810
L2	0.1079	0.0031	0.2967	0.0100	0.0576 / 0.0700

5.3.3 Charging Station Placement

We compute the social cost of charging station placement problem for Singapore with the 2LNQRE human behavior model (learnt from repeated charging games). Meanwhile, we compute the charging station placement with the assumption that EV drivers would form Nash equilibrium ("NE"). Then, we compute the social cost of NE placement with the 2LNQRE human behavior model. The social cost results in Table 5.5 show that by using the 2LNQRE human behavior model in placement problem, we can decrease the social cost by at least 5.25% and as much as 15.51% when the budget is 500.

TABLE 5.5: Social cost comparison

Budget	300	400	500	600
2LNQRE	6947.82	5256.62	4190.56	3523.95
NE	7625.75	5750.31	5001.71	3719.27

5.4 Summary

In this work, we study the bounded rational charging behavior of EV drivers and use it to formulate the EV Charging Station Placement problem (CSPP). There are several contributions of this work. (1) We propose a 2LNQRE Charging behavior model based on the QRE model and the level-k thinking model. The proposed model well captures the irrationality of EV drivers in charging activity. (2) We design a series of user studies to simulate the real-world charging scenarios and collect data from human players. Experimental results based on the data show that our behavior model captures the bounded rational charging behavior of EV drivers. (3) The charging station placement problem is formulated considering the EV drivers' bounded rational charging behavior. An efficient algorithm is designed to solve this complex optimization problem. We show that our approach significantly decreases the social cost. The EV charging behavior model can also be applied to other related problems. For example, when charging stations have been constructed, governors can use pricing as a method to incentivize the EV drivers' charging decision.

Although we carefully design the user studies by abstracting the most important features of the charging scenarios, we have to admit that the laboratory environment is still different from the real-world problems. In the future, when electric vehicles are common, we might need to collect data from real EV drivers and analyze their behavior mode to validate our proposed behavior model.

Chapter 6

Conclusion and Future Work

In this thesis, we present three works for electric vehicle charging station placement and management in sequence. In the following, we would firstly summary the contributions and conclusions from the work we have done. Then we discuss the future work and plan.

6.1 Conclusion

For the first work on charging station placement, we (1) formulate a realistic model for the CSPP in cities like Singapore considering the interactions among charging station placement, EV drivers' charging activities, traffic congestion and queuing time; (2) transform an equivalent single level CSPP from the bi-level CSPP optimization problem obtained through exploiting the structure of the charging game; (3) develop an effective heuristic approach that can speed up the mixed integer CSPP with a large amount of non-linear constraints; (4) conduct theoretical analysis on PoA and corresponding experiments for the charging game; and (5) demonstrate experiments results based on real data from Singapore, which show that our approach solves an effective allocation of charging stations and outperforms baselines.

For the second work on optimal pricing for charging station management, we take a game-theoretic perspective to study the EV charging station pricing problem motivated by the practical need of EV promotion in Singapore. Our first contribution of this work is a novel pricing model that comprehensively incorporates EV users' self-interested charging behavior and their various traffic patterns, traffic congestion contributed by EVs and other non-EV vehicles in the road network, as well as the financial concern for a sustainable operation of the charging network. The second contribution is the algorithm, *SSGA*, to solve the mixed integer non-convex optimal pricing problem, which features two key rules that guarantee efficient converging to equilibrium solution and drastically improves the running time performance. The final contribution is our extensive experiments and results which demonstrate our approach in several aspects, including solution quality, scalability, and robustness. Moreover, we compare our approach with uniform pricing and demonstrate how and to what extent *SSGA* can help with improving the traffic system efficiency and decreasing social cost caused by EV owners' charging behavior. Our approach can be applied in various modern cities like the motivating example Singapore to manage the charging stations in the future. We are actively approaching authorities of Singapore to look for such potential application.

In the third work, we formulate the EV Charging Station Placement problem (CSP-P) with consideration of the charging behavior of EV drivers. We propose an LQRE-Charging behavior model for the EV drivers to capture the EV drivers' irrational charging behaviors. From the human data and the analysis on it, we find that (1) human players rarely consider the influence from others' charging behavior and (2) they make decisions based on stationary factors. With the LQRE-Charging behavior model, we compute the optimal solution for CSPP and compare it with two benchmarks. The experimental results show that our approach significantly outperforms the benchmark in terms of the social cost, the average queuing time the maximum queuing time that the EV drivers would encounter. Our approach provides a better charging station placement, which can improve EV drivers' charging experience. This could be helpful in promoting EVs to the public. The EV charging behavior model can also be applied to other relating problems. For example, when charging stations have been constructed, governors can use pricing as a method to guide the EV drivers.

6.2 Future Work

We propose several efficient approaches regarding electric vehicle charging station placement and management in this thesis. Nevertheless, there remain several opportunities to further extend relating research.

For the electric vehicle charging station placement and management, we look forward to more realistic data when electric vehicles become common in the near future. Especially for the human behavior study regarding the electric vehicle drivers' charging activities. Although we abstract the most important features to simulate the charging scenarios and design user studies, the user studies are different from the real-world scenarios. For example, besides the driving distance and time, drivers might also consider other path conditions (e.g., the road width and the number of turns on the path). These features are unrealistic to capture in simulations because, even if we can present all the details, players can hardly consider all of them in the user study process in the same pattern and extend as they would do in a real-world charging scenario. The lab environment is always distinct from the real world. Therefore, when real human behavior data becomes available, we can extend our human behavior analysis approaches to the dataset and validate the 2LNQRE human behavior model.

Despite electric vehicle charging stations, facility placement and management is a common topic for many other resources in urban city construction. There are many other facilities relating to citizens' daily life and convenience. Form schools, shopping centers, and stadiums to hospitals, all of them are closely bounded up with the public life and recreation. Besides those facilities inside cities, there are also many facilities out of the urban area but still super important. For example, railways that connecting cities together, power stations that support electricity for residents and factories and the signal tower etc. For different facilities, there are different challenges existing. We can extend our approaches in many different directions. Specifically, we discuss one possible extension here for the better life of the aging population.

Appendix A

List of Publications

1. Yanhai Xiong, Jiarui Gan, Bo An, Chunyan Miao, and Ana L. C. Bazzan. Optimal electric vehicle charging station placement. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 2662–2668, 2015

2. Yanhai Xiong, Jiarui Gan, Bo An, Chunyan Miao, and Yeng Chai Soh. Optimal pricing for efficient electric vehicle charging station management. In *Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems*, pages 749–757. International Foundation for Autonomous Agents and Multiagent Systems, 2016

3. Yanhai Xiong, Jiarui Gan, Bo An, Chunyan Miao, and Ana LC Bazzan. Optimal electric vehicle fast charging station placement based on game theoretical framework. *IEEE Transactions on Intelligent Transportation Systems*, 2017

4. Yanhai Xiong, Haipeng Chen, Mengchen Zhao, and Bo An. Hogrider: Champion agent of microsoft malmo collaborative ai challenge. In *Proceedings of the 32nd AAAI Conference on Artificial Intelligence*, 2018

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