Optimal Electric Vehicle Fast Charging Station Placement Based on Game Theoretical Framework

Yanhai Xiong, Jiarui Gan, Bo An, Chunyan Miao, and Ana L. C. Bazzan

***Abstract*— To reduce the air pollution and improve the energy efficiency, many countries and cities (e.g., Singapore) are on the way of introducing electric vehicles (EVs) to replace the vehicles serving in current traffic system. Effective placement of charging stations is essential for the rapid development of EVs, because it is necessary for providing convenience for EVs and ensuring the efficiency of the traffic network. However, existing works mostly concentrate on the mileage anxiety from EV users but ignore their strategic and competitive charging behaviors. To capture the competitive and strategic charging behaviors of the EV users, we consider that an EV user's charging cost, which is dependent on other EV users' choices, consists of the travel cost to access the charging station and the queuing cost in charging stations. First, we formulate the Charging Station Placement Problem (CSPP) as a bilevel optimization problem. Then, by exploiting the equilibrium of the EV charging game, we convert the bilevel optimization problem to a single-level one, following which we analyze the properties of CSPP and propose an algorithm Optimizing eleCtric vEhicle chArging statioN (OCEAN) to compute the optimal allocation of charging stations. Due to OCEAN's scalability issue, we furthermore present a heuristic algorithm OCEAN with Continuous variables to deal with large-scale real-world problems. Finally, we demonstrate and discuss the results of the extensive experiments we did. It is shown that our approach outperform baseline methods significantly.**

***Index Terms*— Electric vehicle charging station, congestion game, facility placement.**

I. Introduction

FOSSIL fuels are generally considered as non-renewable resources and their running out is only a matter of time. Meanwhile, the environment problem caused by burning the fossil fuels is aggregating. Therefore, it has been an arisen topic to study and use alternative energies. Transportation is a main consumer of fossil fuel energy and contributes a large proportion to the pollution. Electric Vehicles (EVs) are promising to replace traditional internal combustion vehicles and move pollution away from urban areas. Electricity be efficiently transformed from both fossil fuels and renewable energies (e.g., solar energy and tidal energy). Thus EVs on the road can achieve zero emission and reduce the pollution from transportation. In recent years, there has been a rapid growth of studies on EVs accompanying with the rising popularity of the smart city concept [1]. A top-priority element for efficient and fast diffusion of EVs is the support of charging facilities like fast charging stations. Although charging at home is an alternative for the EV users, it costs too much time (which can reach 6 to 8 hours). Charging stations with high voltage [2] is then a necessity for the convenience of EV users, because it can charge the EVs at least 12 times faster. The EV drivers' convenience of charging is highly dependent on the distribution of charging stations. Thus the latter can affect the public's willingness of choosing EVs, and the EV drivers' charging behaviors. Consequently the traffic conditions in the road network and the charging system's performance are also influenced.

Among the existing works on EV charging station placement, most of them propose optimization models from different point of views. The optimization objectives include investors' financial cost (construction cost [3], [4] and maintenance cost [5] etc.), EV users' convenience (EV users' access cost [6] and charging station coverage [7], [8] etc.). Others formulate different models for specific problems. For example, the hitting set problem model is used in the work of Funke *et al.* [9], who plan the charging station to ensure energy supply in any shortest path commonly used in the region. A multinomial logit model is employed by He *et al.* [10] to anticipate the EV users' choice distribution among difference charging stations. However, none of existing works manage to comprehensively considered the self-interested charging behavior of the EV drivers. The EV users always prefer to select charging destination and route that can reduce their cost. As a result, their charging behavior can make difference in the traffic condition and the charging system's performance.

To study the Charging Station Placement Problem (CSPP) realistically, we consider the self-interested charging behaviors of EV users, which are competitive and strategic. The interaction of charging behaviors with environment factors including traffic condition in the road network and queuing condition in

Manuscript received May 26, 2016; revised June 30, 2017; accepted September 2, 2017. Date of publication October 10, 2017; date of current version August 1, 2018. This work was supported by the National Research Foundation, Prime Minister's Office, Singapore, under its IDM Futures Funding Initiative. The Associate Editor for this paper was E. Kosmatopoulos. *(Corresponding author: Yanhai Xiong.)*

Y. Xiong is with the Joint NTU-UBC Research Centre of Excellence in Active Living for the Elderly (LILY), Interdisciplinary Graduate School, Nanyang Technological University, Singapore 639798 (e-mail: yanhaixiong7@gmail.com).

J. Gan is with the Department of Computer Science, University of Oxford, Oxford OX1 2JD, U.K. (e-mail: jiarui.gan@gmail.com).

B. An and C. Miao are with the School of Computer Science and Engineering, Nanyang Technological University, Singapore 639798 (e-mail: boan@ntu.edu.sg; ascymiao@ntu.edu.sg).

A. L. C. Bazzan is with the Institute of Informatics, Universidade Federal do Rio Grande do Sul, Porto Alegre 90040-060, Brazil (e-mail: bazzan@inf.ufrgs.br).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TITS.2017.2754382

charging stations are also formulated into the model to decide the optimal charging station placement. There are mainly three reasons for such consideration. First, the queuing condition in charging stations is considered because the queuing experience in charging stations is proven to be significant on the adoption of EVs [11], [12]. Second, inspired by the works of Gan *et al.* [13], [14], we can see that the traffic congestion is influential in car drivers' driving activity especially during peak hours. Thus we plan the charging stations based on the peak hour traffic network to minimize the charging activities' influence on the traffic condition. Third, since the EV users' cannot be centralized, we need to analyze how their charging behaviors are influenced by factors like distribution and size of charging stations, and traffic condition.

Our work makes three main contributions. Firstly, we build a realistic CSPP model, in which the EV drivers' strategic charging behaviors, the traffic condition and the queuing time in charging stations are considered. The overall objective is set as minimizing the total charging cost of EV drivers (named social cost), and EV drivers are assumed to minimize their charging cost with strategic charging behavior. We formulate the CSPP as a bi-level optimization problem, where we take the the social cost as the upper-level objective, which is the goal of the government (who is assumed to be the one to decide the placement of charging stations); a charging game (which falls into the class of congestion games) is formulated in the sub-level problem and Nash Equilibrium is adopted to define the EV drivers' charging behaviors. Secondly, we successfully transfer the bi-level optimization problem into an equivalent single level optimization problem by analyzing the definition and structure of the Nash Equilibrium in the charging game. We propose the algorithm OCEAN (Optimizing eleCtric vEhicle chArging statioN placement) to compute the optimal charging station placement. However, the real-world problems have large scale of variables, and OCEAN is unable to handle them due to the existence of integer variables and the huge variable space. Thus we furthermore work out a heuristic algorithm OCEAN-C (OCEAN with Continuous variables) that can handle the real-world CSPP and ensure solution quality. Thirdly, we design and execute a lot of experiments for both mock data and the real situation of Singapore. The experimental results prove that the designed algorithms OCEAN-C can efficiently solve the CSPP and our approach outperforms some typical baseline methods.

II. Charging Station Placement in Singapore

Named as "Garden City", Singapore has a good reputation for its nice environment and air quality. However, there is still pollution around here, especially the air pollution caused by the heavy traffic that surround us all day. According to the official data [15], 20% of the total carbon emission and 75% of the air pollution in Singapore are caused by the land transportation system, mostly attributed by the motorised traffic. As a result, Singapore government is working on mitigating the environment problem due to the traffic system by introducing the clean EVs as replacement for traditional internal combustion vehicles. The Singapore authorities have started to test the possibility and feasibility of introducing EVs into Singapore since 2011. As a metropolis with advanced energy network, the electric-car manufacturer BYD Asia-Pacific announced that Singapore has the "best potential" to implement EVs [16].

Fig. 1. Zonal Map of Singapore

The construction of EV charging stations is the first challenge, to which the government needs to rise for successful introduction of EVs. Besides the finance concern, there are some elements far more important and urgent, among which traffic condition is of top priority. The planning of charging stations calls for careful investigation to avoid aggravating the traffic congestion of this small city. Specifically, Singapore is a small metropolis with a very small territory of 718.3 km^2 only (Fig. 1). The maximum east west distance is $42km$, while the north south distance is barely $23km$. In a city like Singapore, the most commonly considered problem, namely the limited EVs mileage (usually above $100km$ and some can exceed $500km$ [17]) is not a big issue. Contrasting with the small territory of Singapore is its huge population, which also means a large population of vehicles. According to the Singapore official announcement, there are more than 970 000 motor vehicles in year 2014 on this small island, which indicates the heavy traffic. The fact implies that rather than limited mileage, we should make more efforts on balancing the traffic in consideration of EV users' charging behaviors.

Our first consideration is to minimize the traffic congestion caused by the charging activities in the process of planning the distribution of charging stations. The traffic condition is influenced by charging activities of all the EV drivers. In return, it also influences the EV drivers' decision making of choosing charging destinations. Moreover, the queuing time in charging stations is also studied as an vital element that affects EV drivers' charging decisions. One reason is the long queuing time implies larger space required to accommodate queuing EVs in charging stations. Another is that it would frustrate the EV drivers. We model the interactions among the allocation of charging stations, EV drivers' strategic and self-interested charging activities, traffic congestion on the roads and queuing time in charging stations to formulate the CSPP realistically. To compute the optimal solution, we propose OCEAN and an efficient heuristic algorithm OCEAN-C.

III. Charge Station Placement Problem

To minimize the social cost (defined in Section III-C), we try to find the optimal charging station placement in a region. In the following, we first define the topology of the studied

TABLE I
NOTATION OVERVIEW

Notation	Description
$\mathcal{N} = \{1, ..., n\}$	The set of n zones (i.e., the charging station candidates)
γ_i	The number of resident EV drivers in zone i
x_i	The allocated number of chargers in zone i
B	The budget of chargers to be allocated
$A = \{a_{ij}\}$	The adjacent relationship among zones
$R = \{\langle i, j \rangle\}$	The set of roads with $a_{ij} = 1$
$D = \{d_{ij}\}$	The distance between pairs of zones
α_{ij}^0	The background congestion on road $\langle i, j \rangle$
α_{ij}	The congestion on road from zone i to zone j with consideration of the charging EVs
y_{ij}	The number of EVs from zone i, charge in j
$y_j = \sum y_{ij}$	The total number of EVs charge in zone j
f_{ij}	The travel time cost of an EV $\in y_{ij}$
λ	Parameter in travel cost function
$1/\tau$	The proportion of EVs charge during peak hours
k_{ij}	The inverse of road capacity (used for α_{ij})
μ	The serve capacity per charger per unit time
g_i	The queuing time in charging station i
$\mathbf{p}_i = \{p_{ij}\}$	The charging strategy of EVs in zone i
$\mathbf{P}(\mathbf{P}_{-i})$	The strategy profile of all EVs (except EVs from zone i)
C_i	The charging cost of all EVs in zone i

region, and then explain how we define the charging cost of the EV users. A congestion-game-based interpretation of the CSPP is introduced afterwards, which is followed by a bi-level optimization formulation. For better understand of the definitions, we present all the notations used in problem definition section in Table I.

A. Zones and Charging Stations

We divide the region to be analyzed into n zones in set $\mathcal{N} = \{1, 2, ..., n\}$ according to the geographic and residential condition. We assume each zone is a candidate for building charging station. The specific position of the station can be decided through preliminary studies, which is out of our consideration. For simplification, we name the candidate position as center of the zone. In the following, we also use the set of zones to represent the set of changing station candidates. Any pair of zones are treated as adjacent if they share a geographical border and they are directly connected by a main road. The matrix $A = \{a_{ij}\}_{n\times n}$ is used to represent the adjacency relationship between different zones, where $a_{ij} = 1$ and $a_{ij} = 0$ respectively represents that zone i and zone j are adjacent or nonadjacent. For the ease of notations, we define a zone to be adjacent to itself, i.e., $a_{ii} = 1$. The matrix $D = \{d_{ij}\}_{n\times n}$ denotes the distances between pairs of zones. The average length of trips of EV drivers that reside in zone i and charge in zone j is d_{ij}, which is estimated by the distance between their centers, and d_{ii} is set as the radius of zone i. The concrete example of this paper is Singapore. According to the conventional partitioning method from the official site, we divide it into a number of zones as shown in Fig. 1.

B. The EV Model

Although EVs can be charged at home, some EV drivers would still need charging stations because (1) not everyone has his/her own garage to charge the EV and (2) they might forget to charge during night (since charging at home is time-consuming) and need fast charging. We assume the number of resident EV owners in need of charging in charging stations in zone i as γ_i. The size of the charging station to be built in zone i, i.e., the number of chargers is denoted as x_i, which is to be decided in this work. Note that x_i is integer and can be 0 (meaning that no charging station is built here). Intuitively, EV owners are not willing to drive too far to charge.[1] Thus we assume that EV drivers can choose any one from adjacent zones to charge. The number of EVs that charge in zone i during peak hours is denoted as y_i. Assume that electricity prices are the same in different charging stations, different charging destinations are indifferent in financial cost. Thus we only consider the time cost for EV drivers, including the travel time and the queuing time.

1) Travel Time: We consider the distance d and traffic condition α (i.e., congestion level) on the road as two factors that influence the travel time. The relationship between travel time f and the two factors is shown with Eq. (1), where λ is a constant [18].

$$f_{ij} = \lambda d_{ij} \alpha_{ij} \tag{1}$$

The congestion level α depends on the traffic on the road and is defined in Eq. (2) following transportation science research [19]–[22]. When there are more than one road directly leading from zone i to zone j, we use the average traffic condition, road capacity and distance. We use α_{ij}^0 to denote the background traffic congestion, i.e., the normal traffic congestion caused by any other vehicles except the EVs heading for charging.

$$\alpha_{ij} = \alpha_{ij}^0 + k_{ij} y_{ij}/\tau \tag{2}$$

Note that k_{ij} denotes the inverse proportion of the road capacity; the charging flow from zone i to zone j is represented by y_{ij}; and the fraction of EVs that charge during peak hours is set as $\frac{1}{\tau}$. Thus $k_{ij}\frac{y_{ij}}{\tau}$ represents the congestion caused by EV users' charging activities. The congestion level within zone i is α_{ii}, set as the average congestion level of the main roads inside zone i. We focus on the traffic condition and charging demand during peak hours because (1) the traffic congestion is usually most serious during peak hours period and (2) there are some EV users have to charge during this period due to their limited time and urgent energy demand.

2) Queuing Time: Besides the traffic condition, we also consider EV users' charging activities' influence on the queuing time in charging stations during peak hours. Recalling that we assume that 1 in every τ EVs would charge at charging stations during peak hours, we use $\frac{y_i}{\tau}$ to denote the number of EVs that arrive in zone i for charging during peak hours. We assume that the average queuing time of EV users is

[1] To verify whether the assumption is believable, we relax it and allow EVs to charge in nonadjacent zones in experiments as described in Section V-B.4. The results prove it to be acceptable.

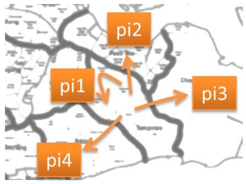

Fig. 2. Strategy demonstration.

directly proportional to the number of EVs in the same station, which can be formally defined as

$$g_i = y_i / \mu \tau x_i. \tag{3}$$

Note that we use μ to denote the serving rate of chargers, i.e., the number of EVs can be served per charger per unit time.

C. A Congestion-Game-Based Interpretation

As we can see from the definition of charging cost in Section III-B, when the background traffic (i.e., the corresponding parameters) and the charging station placement (i.e., the number of chargers in each zone) are fixed, the travel time and queuing time both are decided only by the number of EVs that are using this corresponding road or charging station. We can treat the roads and charging stations as congestible resources. Thus EV users are playing a charging congestion game [23]. We formally define the components of the charging game in the following.

- **Congestible element.** There are two sets of congestible elements in the charging game, respectively the charging stations (i.e., the set of zones), which are denoted as $\mathcal{N} = \{1, ..., n\}$ and the roads (among pairs of adjacent zones and inside each zone) denoted by $\mathcal{R} = \{\langle i, j\rangle | i, j \in \mathcal{N}, a_{ij} = 1\}$. Note that a road $\langle i, j\rangle$ is sensitive to the direction and represents the road leading from zone i to adjacent zone j.
- **Player.** We regard the γ_i EV users in the same zone as identical players with the same strategies.
- **Strategy.** For each player i, we assume that a pure strategy is to charge in a zone j adjacent to zone i,[2] i.e., to use congestible elements charging station i and corresponding road $\langle i, j\rangle$. The players can play mixed strategies, which significant the group of EVs in the same zone charge with different pure strategies and their choices form a distribution. Formally, the probability that EVs in zone i charge in zone j is denoted as p_{ij} and the mixed strategy of player i is defined as $\mathbf{p}_i = \{p_{ij}\}$. For example, a group of EV drivers in zone i can charge in 4 different zones as shown in Fig. 2 Then the strategy profile of all players are denoted as $\mathbf{P} = \langle \mathbf{p}_i \rangle$.
- **Cost**. The congestion cost for each congestible element is defined in Eqs. (1) and (3) respectively for $i \in \mathcal{N}$ and $\langle i, j\rangle \in \mathcal{R}$. For simplicity, we use $g_i(\cdot)$ and $f_{ij}(\cdot)$ to denote the congestion cost, whose variable is the number of users for corresponding congestible element.

[2]This can also be assumed as a set of en-route zones.

According to the players' strategy profile, we can then derive the number of users of each congestible element. For congestible elements $\mathcal{R}$ and $\mathcal{N}$, the number of users under strategy profile $\mathbf{P}$ is respectively:

$$y_{ij} = \gamma_i p_{ij}, \tag{4}$$

$$y_j = \sum\nolimits_{i \in \mathcal{N}} y_{ij}. \tag{5}$$

Next, we can define the charging cost of each player i according to the derived cost for each congestible element. For easy notation, we denote the set of adjacent zones of zone i as $\mathcal{A}_i = \{j | a_{ij} = 1\}$. Then the charging cost of player i, i.e., C_i can be formulated as a function of the strategy profile $\mathbf{P}$ as in the following

$$C_i(\mathbf{P}) = \sum\nolimits_{j \in \mathcal{A}_i} \gamma_i p_{ij} (g_j(y_j) + f_{ij}(y_{ij})). \tag{6}$$

D. Bilevel Optimization Formulation

For the solution concept of the above charging congestion game, we adopt the mixed strategy Nash equilibrium concept. Specifically, with the assumption that all the players are aware of other players' charging strategies, under the equilibrium state, no player can decrease her charging cost by unilaterally changing her own charging strategy. Formally, we can define the equilibrium state with a set of optimizations

$$\mathbf{p}_i \in \arg\min\nolimits_{\mathbf{p}'_i} C_i(\mathbf{P}_{-i}, \mathbf{p}'_i), \quad \forall i \in \mathcal{N}.$$

Note that we use $\mathbf{P}_{-i}$ to denote the strategy profile of players except player i (i.e., type i EVs).

When planning the charging station placement, we stand with the government authority, whose goal is to minimize the social cost when given a fixed budget, a number B of chargers. Consider the overall benefits, we define the social cost as the total charging cost of all players,[3] which can be formally defined as the following formulation when given a charging station placement plan.

$$C(\mathbf{P}) = \sum\nolimits_{i \in \mathcal{N}} C_i(\mathbf{P}). \tag{7}$$

Note that the social cost is a function of the charging strategy of all players, i.e., $\mathbf{P}$, because their strategies influence the charging cost of each of them, and sequentially the social cost.

Considering that the the government authority wants to decide the best charging station placement $\mathbf{x}$ for the minimal social cost regarding to the players' equilibrium in the charging game, we can formulate the CSPP as the following bi-level program $\mathbf{P}$. Eq.(8) is the objective; Eq.(9) is for the budget constraint; Eq.(10) computes the equilibrium strategies of the EV drivers; and the other equations are constraints for the strategies, including the positivity and the 1-sum property. Note that now $C_i(\mathbf{P})$ is also a function of $\mathbf{x}$, but we omitted

[3]We are able to extend our work to handle other kinds of social cost function, like the financial cost.

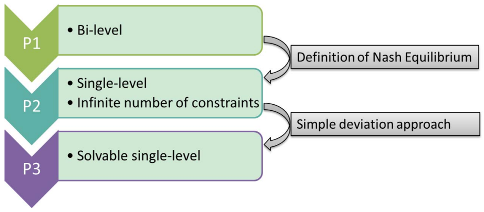

Fig. 3. Approach flow.

that in the expression for simplicity.

$$\textbf{P1:}\ \min_{\mathbf{x},\ \mathbf{P}} C(\mathbf{P}), \tag{8}$$
$$\text{s.t.} \sum_{i\in\mathcal{N}} x_i \le B, x_i \in \mathbb{N}, \tag{9}$$
$$\mathbf{p}_i \in \arg\min_{\mathbf{p}_i'} C_i(\mathbf{P}_{-i}, \mathbf{p}_i'), \quad \forall i \in \mathcal{N}, \tag{10}$$
$$\sum_{j\in\mathcal{A}_i} p_{ij} = 1, \quad \forall i \in \mathcal{N} \tag{11}$$
$$p_{ij} = 0, \forall i \in \mathcal{N}, \quad \forall j \notin \mathcal{A}_i, \tag{12}$$
$$p_{ij} \ge 0, \quad \forall i, j \in \mathcal{N}. \tag{13}$$

We compute the optimal charging station placement with the Nash equilibrium that can achieve the best social cost. When there are multiple Nash equilibria for a placement, the governor can take steps to lead the EV users to form the best equilibrium with the lowest social cost. For example, the governor can provide bounty for some behaviors. Similar idea is studied widely in security games named tie-breaking [24].

IV. Solve the CSPP

After we formulate the charging station placement problem as a bi-level optimization problem **P1**, we focus on the algorithm to solve it. The flow of our approach is presented with Fig. 3. From problem **P1** we can see that the sub-level optimization has multiple objectives, each of which is the objective for a type of EV users in the charging game. This feature makes the problem complicated and unable to be handled with existing solvers. Therefore, we first work on the sub-level optimization problem (Eqs. (10) – (13)) and propose an efficient approach to transfer the sub-level optimization problem into a number of constraints, which can restrict the Nash equilibrium space of the charging game (i.e., the solution of the sub-level optimization problem). Then we can result in an equivalent single-level optimization, which is still difficult due to the large number of variables (including integer variables and continuous variables) and large searching space of the integer variables. We propose a searching algorithm for the single-level optimization problem to speed up the computation. Next, we start with analyzing the Nash equilibrium criterion in formulation, which is useful for problem reformulation.

A. Deviation of Strategies

According to the definition of equilibrium that we mentioned in last section, we need to consider the player's unilateral strategy change and the influence in its charging cost to prove the stableness of the equilibrium state. Here we use an n-dimensional vector $\Delta\mathbf{p} = (\Delta_1, ..., \Delta_n)$ to denote player i's unilateral strategy change with reference to the strategy profile $\mathbf{P}$. The strategy change is named strategy deviation and should meet the following criterions.

$$\sum_{j\in\mathcal{N}} \Delta_j = 0 \tag{14}$$
$$-p_{ij} \le \Delta_j \le 1 - p_{ij}, \quad \forall j \in \mathcal{A}_i \tag{15}$$

When type i players change their strategy from $\mathbf{p_i}$ to $\mathbf{p_i}' = \mathbf{p_i} + \Delta\mathbf{p}$, recall that y_{ij} denotes the charging flow from zone i to zone j and y_j denotes the number of EVs that charge in zone j, we have $y_{ij}' = y_{ij} + \gamma_i\Delta_j$, $y_j' = y_j + \gamma_i\Delta_j$, and the change in type i EVs' cost can be formulated as:

$$\begin{aligned}\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) &= C_i(\mathbf{P}_{-i}, \mathbf{p_i}') - C_i(\mathbf{P})\\ &= \sum_{j\in\mathcal{A}_i} \gamma_i[p_{ij}(\lambda d_{ij}k_{ij}\frac{\gamma_i\Delta_j}{\tau} + \frac{\gamma_i\Delta_j}{\mu\tau x_j})\\ &\quad + \Delta_j(\lambda d_{ij}\alpha_{ij} + \lambda d_{ij}k_{ij}\frac{\gamma_i\Delta_j}{\tau} + \frac{y_j}{\mu\tau x_j} + \frac{\gamma_i\Delta_j}{\mu\tau x_j})]\\ &= \sum_{j\in\mathcal{A}_i} \gamma_i[(\frac{p_{ij}\gamma_i}{\tau}(\lambda d_{ij}k_{ij} + \frac{1}{\mu x_j}) + \lambda d_{ij}\alpha_{ij} + \frac{y_j}{\mu\tau x_j})\Delta_j\\ &\quad + (\lambda d_{ij}k_{ij}\frac{\gamma_i}{\tau} + \frac{\gamma_i}{\mu\tau x_j})\Delta_j^2].\end{aligned} \tag{16}$$

For the ease of description, we rewrite it as

$$\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = \sum_{j\in\mathcal{A}_i} \gamma_i(\xi_{ij}\Delta_j + \eta_{ij}\Delta_j^2). \tag{17}$$

We can reformulate the CSPP **P1** with the Nash equilibrium definition – no player has the incentive to deviate.

$$\textbf{P2:}\ \min_{\mathbf{x},\mathbf{P}} C(\mathbf{P}), \tag{18}$$
$$\begin{aligned}&\text{s.t. } \Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \ge 0, \quad \forall i \in \mathcal{N}, \quad \forall\Delta\mathbf{p},\\ &(9), (11)\text{–}(13).\end{aligned} \tag{19}$$

We use Eq. (19) to restrict the Nash equilibrium space instead of using Eq. (10). In this case, we have reformulated the bi-level optimization problem into a single-level one. However, there are infinite number of constraints in the problem, because $\Delta\mathbf{p}$ of Eq. (19) for each i is a vector with continuous elements. Thus we need to furthermore find a way to solve the problem **P2**. We propose a *simple deviation approach*, which can replace Eq. (19) with a finite number of constraints and make the optimization problem solvable.

B. Simple Deviation Approach

Before introducing the approach, we first define a special type of deviation called *simple deviation*.

Definition 1 (Simple Deviation): A simple deviation of type i player is a strategy change, where only the probabilities of a pair of pure strategies are changed (one increases and the other decreases by the same amount), while the probabilities of all the other pure strategies remain unchanged. A simple deviation is denoted as a tuple $\langle l, h, \delta\rangle$ with $\delta > 0$, which corresponds to a deviation vector $\Delta\mathbf{p}$, such that $\Delta_l = -\delta$, $\Delta_h = \delta$, and $\Delta_j = 0, \forall j \notin \{l, h\}$.

We can then prove an important property of CSPP as Lemma 1 based on *simple deviation*, which is used for simplifying the equilibrium criterion in the derived program **P2**.

Lemma 1: Given a strategy profile $\mathbf{P}$ *with* $p_{il} > 0$*, type* i *player cannot reduce her charging cost through a unilateral simple deviation from pure strategy* l *to* h *(i.e., reduce* p_{il} *and increase* p_{ih}*), if and only if* $\xi_{ih} \geq \xi_{il}$.

Proof: The basic idea to prove this lemma is to derive the charging cost change due to an unilateral simple deviation and analyze it. From Definition 1 and Eq. (17), we can see the charging cost change due to an unilateral simple deviation $\langle l, h, \delta \rangle$ for type i players can be denoted as

$$\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) = \gamma_i(\eta_{il} + \eta_{ih})\delta^2 + \gamma_i(\xi_{ih} - \xi_{il})\delta. \quad (20)$$

Note that this is a quadratic function of δ. While player i with a nonzero simple deviation $\Delta\mathbf{p} = \langle l, h, \delta \rangle$ cannot reduce her charging cost, we have $p_{il} > 0$ and $\delta \in [0, p_{il}]$; what we need to prove is $\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \geq 0$. From Eq. (16) we can easily get $\eta_{il} + \eta_{ih} > 0$. As a result, ξ_{ih} has to be no smaller than ξ_{il} to ensure $\Delta C_i(\mathbf{P}, \Delta\mathbf{p})$ to be non-negative for all possible value of δ. We can show this with discussion in two cases. First, if $\xi_{ih} < \xi_{il}$, there is always some $\delta < \frac{\xi_{il} - \xi_{ih}}{\eta_{il} + \eta_{ih}}$ such that $\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) < 0$. Second, if $\xi_{ih} \geq \xi_{il}$, we can easily see that $\Delta C_i \geq 0$ for all $\delta \geq 0$. Therefore, type i player with $p_{il} > 0$ cannot reduce her charging cost through a simple deviation from pure strategy l to h if and only if $\xi_{ih} \geq \xi_{il}$. □

Lemma 2: If a player cannot reduce her cost by any unilateral simple deviation, then she can neither reduce her cost by any unilateral strategy deviation.

Proof: Before proving the lemma, we show that an arbitrary unilateral strategy deviation $\Delta\mathbf{p}_i$ for any player i can be decomposed into a number of unilateral simple deviations, thus charging cost change $\Delta\mathbf{p}_i$ can also be decomposed. For simplicity, we denote the unilateral strategy deviation of player i as $\Delta\mathbf{p} = (\Delta_1, ..., \Delta_n)$. For the elements in the vector $\Delta\mathbf{p}$, there must be negative and positive ones, for which we use two sets $\mathcal{L} = \{i | i \in \mathcal{N}, \Delta_i < 0\}$ and $\mathcal{H} = \{i | i \in \mathcal{N} \Delta_i > 0\}$ to represent respectively. We can see that implement of deviation $\Delta\mathbf{p}$ can be achieved by a number of simple deviations, where each is a deviation from an $l \in \mathcal{L}$ to an $h \in \mathcal{H}$ with the proportion $\delta_{hl} = |\Delta_l| \cdot \frac{\Delta_h}{\sum_{i \in \mathcal{H}} \Delta_i}$.

Consequently, we decompose the change in charging cost due to an arbitrary strategy deviation as in the following.

$$\begin{aligned}
\frac{\Delta C_i(\mathbf{P}, \Delta\mathbf{p})}{\gamma_i} &= \sum\nolimits_{j \in \mathcal{A}_i} (\xi_{ij}\Delta_j + \eta_{ij}\Delta_j^2) \\
&= \sum\nolimits_{l \in \mathcal{L}} \big(\xi_{il}(-\sum\nolimits_{h \in \mathcal{H}} \delta_{hl}) + \eta_{il}(-\sum\nolimits_{h \in \mathcal{H}} \delta_{hl})^2\big) \\
&\quad + \sum\nolimits_{h \in \mathcal{H}} \big(\xi_{ih}(\sum\nolimits_{l \in \mathcal{L}} \delta_{hl}) + \eta_{ih}(\sum\nolimits_{l \in \mathcal{L}} \delta_{hl})^2\big) \\
&\geq \sum\nolimits_{l \in \mathcal{L}} \big(\xi_{il}(-\sum\nolimits_{h \in \mathcal{H}} \delta_{hl}) + \eta_{il}(\sum\nolimits_{h \in \mathcal{H}} \delta_{hl}^2)\big) \\
&\quad + \sum\nolimits_{h \in \mathcal{H}} \big(\xi_{ih}(\sum\nolimits_{l \in \mathcal{L}} \delta_{hl}) + \eta_{ih}(\sum\nolimits_{l \in \mathcal{L}} \delta_{hl}^2)\big) \\
&= \sum\nolimits_{l \in \mathcal{L}} \sum\nolimits_{h \in \mathcal{H}} (\eta_{il} + \eta_{ih})\delta_{hl}^2 + (\xi_{ih} - \xi_{il})\delta_{hl}
\end{aligned}$$

Note that for the ease of presentation, the cost change is divided by the number of EV users in zone i, i.e., γ_i. As we can see from the above formulations, the charging cost change due to an arbitrary strategy deviation can be compared with the sum of the charging cost change due to the set of simple deviations that equal to the original deviation and it is always no smaller than the latter. With the prerequisite of the lemma, we know that player i cannot reduce his charging cost by any simple deviation, including the set of simple deviations we had as a decomposition of the arbitrary strategy deviation. Referring to Lemma 1, we can know $\xi_{ih} \geq \xi_{il}$ is true for all $l \in \mathcal{L}$ and $h \in \mathcal{H}$, i.e., the part to be summed in the right hand side of the above inequality is non-negative. Thus we have proved that $\Delta C_i(\mathbf{P}, \Delta\mathbf{p}) \geq 0$. Since $\Delta\mathbf{p}$ and i are arbitrary, thus no player can reduce her charging cost by any unilateral strategy deviation while they cannot achieve that with any unilateral simple deviation. □

Proposition 3: A strategy profile $\mathbf{P}$ *forms a Nash equilibrium if and only if* $\xi_{ih} \geq \xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{il} > 0$.

Proof: The proposition is quite straightforward if we follow Lemma 1, Lemma 2 and the converse direction of Lemma 2, which must hold because a simple deviation is a special case of arbitrary strategy deviation. Under the equilibrium definition, no player can decrease the charging cost with arbitrary unilateral strategy deviation $\Leftrightarrow$ no player can decrease his charging cost using any unilateral simple deviation $\Leftrightarrow \xi_{il} \leq \xi_{ih}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{ij} > 0$. □

Based on Proposition 3, we can avoid an infinite number of non-linear constraints as Eq. (19). With the results from Proposition 3, we know that under the equilibrium strategy profile $\mathbf{P}$, there is $\xi_{ih} \geq \xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i, p_{il} > 0$, which can be reformulated as $p_{il}\xi_{ih} \geq p_{il}\xi_{il}, \forall i \in \mathcal{N}, \forall l, h \in \mathcal{A}_i$. Therefore, we propose OCEAN (Optimizing eleCtric vEhicle chArging statioN placement) in program **P3** to compute the optimal solution of the CSPP instead of using program **P2**.

$$\begin{aligned}
\textbf{P3:}\quad & \min\nolimits_{\mathbf{x},\mathbf{P}} C(\mathbf{P}), && (21)\\
& \text{s.t. } p_{il}\xi_{ih} \geq p_{il}\xi_{il}, \quad \forall i \in \mathcal{N}, \quad \forall l, h \in \mathcal{A}_i, \\
& (9), (11)\text{–}(13). && (22)
\end{aligned}$$

The above program is a single-level non-linear optimization problem and can be handled by a standard non-linear optimization solver.

C. Problem Analysis

An important concept in game theory is the price of anarchy (PoA) [25], which is the ratio between the maximum social cost among different equilibria and the minimum social cost regardless of players' selfish behavior (in other words, assuming the players follow the instruction of a central controller who aims to minimize the social cost). PoA is a concept that measure the worst-case inefficiency of the system caused by the selfish behavior of players. We use S and E to respectively denote the strategy space and Nash equilibrium strategy space of the charging game. They can be formally defined as

$$S = \{\mathbf{P} | \mathbf{P} \text{ satisfies Eqs. (11)–(13)}\}, \quad (23)$$
$$E = \{\mathbf{P} | \mathbf{P} \text{ satisfies Eqs. (11)–(13), (22)}\}. \quad (24)$$

Then the definition of PoA is

$$\text{PoA} = \max_{\mathbf{P} \in E} C(\mathbf{P})/Opt, \quad (25)$$

where Opt denotes the socially optimal cost assuming that all EVs' charging behavior can be controlled, which is

$$Opt = \max_{\mathbf{P}\in S} C(\mathbf{P}). \tag{26}$$

We can prove the theoretical result of PoA as in the following theorem.

Theorem 1: The price of anarchy of the charging game is at most $\frac{3+\sqrt{5}}{2} \approx 2.618$.

Proof: For the ease of description, we first rewrite the linear cost functions (i.e., travel cost and queuing cost) as $c_e(f_e) = a_e f_e + b_e$ for each congestion element $e \in \mathcal{N} \bigcup \mathcal{R}$. According to Eqs. (1) to (3), we have

$$a_e = \begin{cases} \lambda d_{ij} k_{ij} \dfrac{1}{\tau}, & e = \langle i, j\rangle \in \mathcal{R};\\ \dfrac{1}{\mu\tau x_i}, & e = i \in \mathcal{N}; \end{cases}$$
$$b_e = \begin{cases} \lambda d_{ij} k_{ij} \alpha_{ij}^0, & e = \langle i, j\rangle \in \mathcal{R};\\ 0, & e = i \in \mathcal{N}. \end{cases}$$

Obviously, $a_e > 0$ and $b_e \geq 0$. Let $\mathbf{P}$ be a Nash equilibrium strategy, and $\mathbf{P}^*$ be the strategy profile for social optimum. Suppose in a Nash equilibrium, player i deviates by playing the social optimal strategy $\mathbf{p}_i^*$, it follows that

$$\begin{aligned}\sum\nolimits_{j\in\mathcal{A}_i} p_{ij} \sum\nolimits_{e\in\mathcal{S}_{ij}} c_e(f_e) &\leq \sum\nolimits_{j\in\mathcal{A}_i} p_{ij}^* \sum\nolimits_{e\in\mathcal{S}_{ij}} c_e(f_e^*)\\ &\leq \sum\nolimits_{j\in\mathcal{A}_i} p_{ij}^* \sum\nolimits_{e\in\mathcal{S}_{ij}} c_e(f_e + f_e^*).\end{aligned}$$

The first inequality holds since $\mathbf{P}$ forms a Nash equilibrium, thus player i can never decrease his charging cost by unilaterally deviating his own strategy.

Since the above inequality holds for all player i, we have

$$\begin{aligned} C(\mathbf{P}) &= \sum\nolimits_{i\in\mathcal{N}} \gamma_i \sum\nolimits_{j\in\mathcal{A}_i} p_{ij} \sum\nolimits_{e\in\mathcal{S}_{ij}} c_e(f_e)\\ &\leq \sum\nolimits_{i\in\mathcal{N}} \gamma_i \sum\nolimits_{j\in\mathcal{A}_i} p_{ij}^* \sum\nolimits_{e\in\mathcal{S}_{ij}} c_e(f_e + f_e^*)\\ &= \sum\nolimits_{i\in\mathcal{N}} \gamma_i \sum\nolimits_{j\in\mathcal{A}_i} p_{ij}^* \sum\nolimits_{e\in\mathcal{S}_{ij}} \left[c_e(f_e^*) + a_e f_e\right]\\ &= C(\mathbf{P}^*) + \sum\nolimits_{i\in\mathcal{N}} \gamma_i \sum\nolimits_{j\in\mathcal{A}_i} p_{ij}^* \sum\nolimits_{e\in\mathcal{S}_{ij}} a_e f_e\\ &= C(\mathbf{P}^*) + \sum\nolimits_{e\in\mathcal{N}\bigcup\mathcal{R}} a_e f_e f_e^*. \end{aligned}$$

We apply the Cauchy-Schwarz inequality to the last term and get following inequality:

$$\begin{aligned} &\sum\nolimits_e a_e f_e f_e^*\\ &\quad\leq \sqrt{\sum\nolimits_e a_e f_e^2} \cdot \sqrt{\sum\nolimits_e a_e (f_e^*)^2}\\ &\quad\leq \sqrt{\sum\nolimits_e f_e \cdot\ (a_e f_e + b_e)} \cdot \sqrt{\sum\nolimits_e f_e^* \cdot\ (a_e f_e^* + b_e)}\\ &\quad= \sqrt{C(\mathbf{P})} \cdot \sqrt{C(\mathbf{P}^*)}. \end{aligned}$$

It follows that

$$\begin{aligned} &C(\mathbf{P}) \leq C(\mathbf{P}^*) + \sqrt{C(\mathbf{P})} \cdot \sqrt{C(\mathbf{P}^*)}\\ &\Rightarrow \frac{C(\mathbf{P})}{C(\mathbf{P}^*)} \leq 1 + \sqrt{\frac{C(\mathbf{P})}{C(\mathbf{P}^*)}}\\ &\Rightarrow 0 \leq \sqrt{\frac{C(\mathbf{P})}{C(\mathbf{P}^*)}} \leq \frac{1+\sqrt{5}}{2} \quad \text{(by solving } x^2 - x - 1 \leq 0)\\ &\Rightarrow \frac{C(\mathbf{P})}{C(\mathbf{P}^*)} \leq \frac{3+\sqrt{5}}{2} \approx 2.618. \end{aligned}$$

Thus we can conclude that the PoA is at most around 2.618. Note that the value 2.618 holds for any charging station placement $\mathbf{x}$. Therefore we can rewrite it more accurately as $\text{PoA} = \max_{\mathbf{x}} \max_{\mathbf{P}\in E} \frac{C(\mathbf{P})}{Opt}$. □

Furthermore, with the formulation **P3** that we derived in previous section, we can compute the PoA for a specific setting in practice as follows, which could be much lower than 2.618. Note that when we compute the optimal solution of the charging station placement problem with **P3**, we are actually computing the charging station placement $\mathbf{x}^*$ with the best minimum equilibrium social cost, i.e.,

$$\mathbf{x}^* \in \arg_{\mathbf{x}} \min_{\mathbf{x},\mathbf{P}\in E} C(\mathbf{P}).$$

Then, we can compute the POA for this specific setting based on charging station placement $\mathbf{x}^*$. We compute the maximum equilibrium social cost $\max_{\mathbf{P}\in E} C(\mathbf{P})$ and social optimum Opt respectively with following programs **P4** and **P5**.

$$\begin{aligned} \textbf{P4:}\quad & \max_{\mathbf{P}}\ C(\mathbf{P}), & (27)\\ & \text{s.t. } \mathbf{x} = \mathbf{x}^*, & \\ & \quad (11)\text{–}(13),\ (22). & (28)\\ \textbf{P5:}\quad Opt &= \min_{\mathbf{P}} C(\mathbf{P}), & (29)\\ & \text{s.t. } \mathbf{x} = \mathbf{x}^*, & \\ & \quad (11)\text{–}(13). & (30) \end{aligned}$$

Note that to compute the optimal social cost without considering the EVs' selfish driving behavior, we eliminate conditions represented by Eq. (22) in **P5**. As we will show later in the experiment section, the computed POA for specific settings is much smaller than 2.618.

D. Speeding Up OCEAN

As we can see from the formulation of OCEAN in **P3**, it is a mixed integer non-linear problem and the number of non-linear constraints expressed in Eq. (22) grows very fast with the number of players and strategies increasing. As a result, OCEAN is unable to handle large-scale real-world problems.

To handle large-scale problems, we compute the optimal solution in two steps by using a heuristic algorithm OCEAN-C (namely OCEAN with Continuous variables), which is shown in Algorithm 1.

Firstly, we relax $\mathbf{x}$ to be continuous variables and solve the optimal solution $\mathbf{x}^*$ of **P3**. Since the number of chargers in $\mathbf{x}^*$ of different zones are not integers, we round $\mathbf{x}^*$ to $\widehat{\mathbf{x}}$. The rounding process is first to take the floor value of each x_i^*, sort the zones according to the $x_i - \lfloor x_i \rfloor$ value descendingly,

Algorithm 1 OCEAN-C

1 Relax $\mathbf{x}$ to be continuous;
2 Solve optimal solution $\mathbf{x}^*$ of **P3**;
3 $\widehat{\mathbf{x}} \leftarrow$ rounded $\mathbf{x}^*$;
4 Compute the optimal solution Obj of **P3** with $\mathbf{x}$ set as $\widehat{\mathbf{x}}$ (refer to Algorithm 2);
5 **return** Obj, $\widehat{\mathbf{x}}$;

Algorithm 2 Sub-OCEAN-C

1 Initiate indicator vector ϕ as $\{0\}$;
2 Set $f_{ij}^0 = \lambda d_{ij}(\alpha_{ij}^0 + k_{ij})$ and $g_i = \frac{1}{\tau x_i}$ for all roads and charging stations with $x_i > 0$;
3 **for** $i \in \mathcal{N}$ **do**
4   Set $c_i^{min} = \min_{j \in \mathcal{A}_i} c_{ij}^0 = \min_{j \in \mathcal{A}_i}(f_{ij}^0 + g_j^0)$;
5   **for** $j \in \mathcal{A}_i$ *with* $x_j > 0$ **do**
6   **if** $c_{ij}^0 \le \varphi c_i^{min}$ **then** Let $\phi_{ij} = 1$;
7 Set $flag = 1$;
8 **repeat**
9   Solve problem **P6** and get objective value Obj;
10   Set $flag = 0$;
11   **for** $i \in \mathcal{N}$ **do**
    /* ——- Rule A ——- */
12   **for** $k \in \mathcal{A}_i$ *with* $\phi_{ik} = 1$ **do**
13   **if** $p_{ik} < 1.0e-6$ **then**
14   Let $\phi_{ik} \leftarrow 0$;
15   Set $flag = 1$;
    /* ——- Rule B ——- */
16   Get a ξ_{ij} with $\phi_{ij} = 1$;
17   **for** $k \in \mathcal{A}_i$ *with* $\phi_{ik} = 0$ **do**
18   **if** $\xi_{ik} < \xi_{ij}$ **then**
19   Let $\phi_{ik} \leftarrow 1$;
20   Set $flag = 1$;
21 **until** $flag = 0$;
22 **return** Obj;

then set $\widehat{x}_i$ for the top $R = B - \sum_{i \in \mathcal{N}} \lfloor x_i^* \rfloor$ zones as $\lfloor x_i \rfloor + 1$ and otherwise $\lfloor x_i \rfloor$. To compute the optimal solution of CSPP, we set $\mathbf{x}$ as $\widehat{\mathbf{x}}$, the result of which is the output of OCEAN-C. With $\mathbf{x}$ determined, the single level CSPP's runtime sharply decreases.

Furthermore, we specify the sub-algorithm of OCEAN-C in Algorithm 2, which is designed to compute the equilibrium of the charging game with a given charging station placement. As we can see from **P3**, the problem is non-linear, and the main difficulty comes from distinguishing the employed pure strategies (with using probability > 0) from the abandoned ones (with using probability $= 0$), which results in constraint denoted by Eq. (22). Then we naturally consider to specify the employed strategies (also named "support") before solving the equilibrium. Following the idea, we design Algorithm 2 to compute the equilibrium, where we first initiate the support manually and gradually expand the support set by carefully comparing the pure strategies until an equilibrium is reached. For a given support set, we use the following program to compute the equilibrium.

$$\textbf{P6:} \quad \min_{\mathbf{P}} \; C(\mathbf{P}), \tag{31}$$

$$\text{s.t.} \quad p_{ij} = 0, \quad \forall i \in \mathcal{N}, \phi_{ij} = 0, \tag{32}$$

$$\xi_{ih} \ge \xi_{il}, \; \forall i \in \mathcal{N}, \; \forall l, h \in \mathcal{A}_i, \phi_{ih} = \phi_{il} = 1,$$

$$(9), (11)\text{–}(13). \tag{33}$$

Note that the vector ϕ is an artificial indicator corresponding to the variable $\mathbf{P}$. When $\phi_{ij} = 0$, we force p_{ij} as 0; if $\phi_{ij} = 1$, then $p_{ij} > 0$ and the corresponding pure strategy is in the support. We only compare the ξ value for strategies in the support set to avoid the problem being infeasible when there is a pure strategy who is not in the support set but its corresponding ξ value is smaller. The problem **P6** is a convex optimization problem with linear constraints that can be solved efficiently.

In Lines 1–6 of the algorithm, we initiate the indicator for each pure strategy according to the *basic* charging cost calculated by assuming that only one player uses the corresponding strategy (as in Line 2) and put some of the pure strategies into the support by comparing the *basic* charging cost. Note that the coefficient ψ in Line 6 is to decide the size of the initial support set and derived from practice. *Rule A* and *Rule B* are two criterions for updating the support space. *Rule A* is used to delete the useless strategies and *Rule B* is for adding better pure strategies into the support. When no change is made after checking the two rules, the algorithm terminates with an equilibrium of the charging game.

V. Experimental Evaluation

In this section, we run experiments on the real data set from Singapore to evaluate our approach. To compare multiple methods, all experiments were run on the same data set using a $3.4GHz$ Intel processor with $16GB$ of RAM, employing KNITRO (version 9.0.0) for nonlinear programs. The results were averaged over 20 trials.

A. Data Set and Baseline Methods

The population of all motor vehicles in Singapore has reached 969 910 in year 2012 according to the statistics in the official websites of Singapore Land Transport Authority (LTA) and Singapore Department of Statistics (DOS). Based on the conventional partition method as shown in Fig. 1, combined with the accessible graphical and residential distribution data on the websites, we divide Singapore into 23 zones to test our approach. A basic assumption is that the number of vehicles is proportional to the number of residents in each zone. Then we assume that 10% among all the vehicles in Singapore are EVs, 5% among which would need charging in charging stations during peak hours. Using the distance measure tool in Google Maps, the distances between adjacent zones' centers are estimated; a normal congestion α_{ij}^0 during the peak hours is taken with the ratio of travel time during peak hours and the distance between zones i and j. The road capacity of the roads

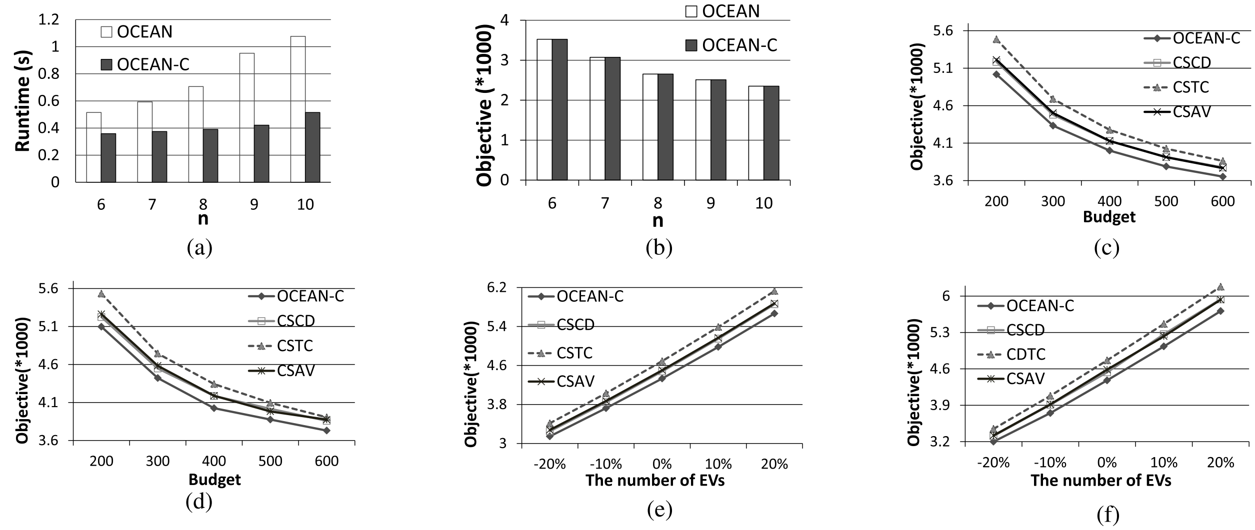

Fig. 4. Compare OCEAN-C with OCEAN and baselines. Figs.a & b are the runtime and optimal objective value of algorithm OCEAN and OCEAN-C when problem size increases; Fig.c is optimal objective values of OCEAN-C and baseline methods when the budget increases, while Fig.d is the corresponding results with human behavior uncertainty; Figs.e and f are results with different number of EV drivers in the charging game.

between any two zones are set as the same value, which means $k_{ij} = 0.01$ for all pairs i and j. We assume averagely 6 EVs can be served in one hour by each charger, i.e., serving rate of chargers is set as $\mu = 6$. The proportion of EVs that charge during peak hours is set as $\frac{1}{\tau} = \frac{1}{10}$. The linear coefficient λ in the travel cost function is fixed at 0.2. Unless otherwise specified, we use the above parameters in all our experiments. We combine some small zones of the 23-zones to generate data of different n (from 6 to 10), so that we can run OCEAN, which has scalability issues, to get the results (both runtime and solution quality) and compare them with OCEAN-C.

To demonstrate the performance of our approach, we compare it with three baseline methods:

- The first baseline method is named CSCD. CSCD assigns the number of chargers to each zone proportional to the number of residential EV users in each zone. Specifically, $x_i \propto \gamma_i$.
- The second baseline method is named CSTC. CSTC assigns the number of chargers in each zone according to the traffic condition as well as the physical distance. Specifically, for each zone i and one of its adjacent zone j, we calculate the reciprocal of $\alpha^0_{ji} d_{ji}$ (intuitively, this value means the difficulty for EV users in zone j to charge in zone i), then sum that value of all adjacent zones together. We decide the number of chargers in zone i as $x_i \propto \sum_{j \in \mathcal{A}_i} 1/(\alpha^0_{ji} d_{ji})$.
- The third baseline method is named CSAV. CSAV assigns the chargers in different zones averagely.

We get the results (the optimal social cost) for each baseline method by first compute the number of chargers in each zone according to the principals described above, then compute the EV users' charging activity equilibrium and the resulted social cost. The program is the same as program **P3** but **x** is fixed rather than a variable.

B. Performance Evaluation

1) OCEAN-C vs. OCEAN: We combine some small zones of the 23-zone division (shown in Fig. 1) to get smaller zone divisions (n changes from 6 to 10) because OCEAN cannot handle large-scale problems. The budget of total number of chargers is set as 300 in the experiments. In Figs. 4a, the runtime performance of OCEAN and OCEAN-C are compared with bars. We can see that OCEAN-C always saves runtime comparing to OCEAN. Moreover, when the problem scale grows, the runtime of OCEAN increases faster then that of OCEAN-C, which indicates that OCEAN-C is much more time-efficient than OCEAN. When we look at the solution quality (i.e., the optimal social cost) depicted in 4b, we can find that OCEAN-C can save runtime without seriously sacrificing the solution quality because the minor difference in social cost of both approach is invisible when expressed with the bars. Therefore, we use OCEAN-C as a substitute approach for OCEAN in following experiments.

2) OCEAN-C vs. Baseline Methods: We compare our approach OCEAN-C with three baseline methods when the number of zones n is set as 23. As we can see from Fig. 4c, when the budget is increasing (from 200 to 600), the optimal objective value of all approaches keeps going down, because more resources usually means better service and customer convenience. Nevertheless, our approach outperforms all of them and achieves minimal social cost. In Fig. 4e, the results of changing the number of EV users are depicted. We can see that when the number of EV users is more, the minimal social cost is higher, because they have more influence on the traffic congestion and also the queuing condition in charging stations. In this case, our approach that takes into account the EV users' strategic behaviors can significantly decrease the social cost. In conclusion, OCEAN-C outperforms the baseline methods.

3) Robustness Evaluation: We then evaluate the robustness of our approach and compare its performance with the baseline methods regarding to the EV users' limited rationality. We assume that EV users are full informative and rational in the problem model. While people might be able to learn the equilibrium in repeated charging activities, there can be some special cases that change their activity in practice. For example, they might need to deal with a special thing or meet

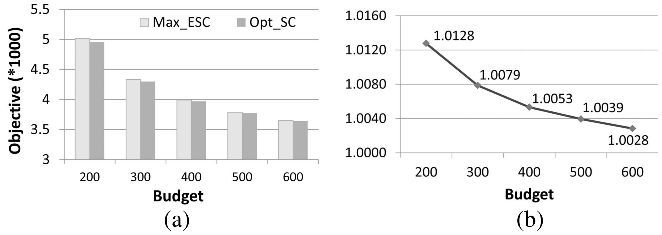

Fig. 5. (a) Compare maximum equilibrium social cost and social optimal; (b) Trend of PoA under different budgets.

someone, which may result in strategy deviation. We assume that there are part of EV users deviate their charging activities from equilibrium, This proportion is set as 10% for each zone, i.e., we compute the social cost again with the 90% of EV users following the equilibrium and 10% of them choose a charging strategy from their strategy space randomly. In Figs. 4d and 4f, we present the robustness test results for all approaches in consideration of different budget and different number of EV users respectively. The number of zones is set $n = 23$. Under comparison with Figs. 4c and 4e, we can see that the EV users' deviation from equilibrium can cause more social cost. However, our approach OCEAN-C can keep the superiority comparing to the baseline methods.

4) EVs Charge in Remote Zones: When we formulate the charging game previously, we made an assumption that the EV users only charge in adjacent zones (including their residential zone). To prove that this assumption is realistic and reasonable, we use experiments to show that almost all EV users only charge in adjacent zones even when they are allowed to charge further, because the latter usually result in higher charging cost. We relax the assumption for EV users in zone i by allowing them to charge in a neighbor zone of its adjacent zones (but not in $\mathcal{A}_i$), which is name two-stop remote zones. We compare the results of the original model and the new one under the same data set. It turns out that the social cost increases slightly, but the change is less than 0.001, which is negligible comparing to the original optimal social cost at about 4000). Moreover, the EV users seldom use the two-stop strategies. As a result we can see that the assumption of charging only in adjacent zones is realistic and reasonable.

5) Experimental Results of PoA: We conduct experiments based on optimal charging station placement derived from OCEAN-C and the experiment set with $n = 23$. The coefficient φ used for initiating the support set in Algorithm 2 Line 6 is set as 1.5. Actually the coefficient can vary in a big range and still work. When it is getting smaller, the number of iterations of solving problem **P6** can increase; and when it is too large, it is possible that the problem becomes infeasible. We select 1.5 as φ value in this set of experiment. The maximum equilibrium social cost and the minimum social cost without consideration of EVs' selfish charging behavior are respectively computed with programs **P4** and **P5**.

As we can see from following Fig. 5a, there is small difference between the maximum equilibrium social cost and optimal social cost respectively depicted by the "Max_ESC" and "Min_SC" bars. We can refer to Fig. 5b for the trend of PoA w.r.t. the budget. From the figure we can see that when the amount of social resource (i.e., the budget for charging station construction here) increases, the inefficiency of the charging system caused by selfish behavior is becoming smaller.

VI. Related Work

In the past years, the raising concern of the shortage of non-renewable energy has made new energy a hot research topic. In the transportation domain, the EV is regarded as an ideal substitute for traditional vehicles.

Many researchers have made efforts in related techniques to enable/speed up the EV diffusion, for example analyzing the key factors that may infect the construction of EV infrastructure [26]. Meanwhile, many researchers are working on integrating EVs into the traditional transportation network, for example with a system for EV integration with energy grid [27]. Rigas *et al.* gave a survey of such researches [28].

While charging is a premium issue for EV diffusion, placement of charging stations and charging mechanism are two important topics worth studying. There are some works studying the charging mechanism/pattern based on settled charging network. Rei *et al.* presented a charging control mechanism for EVs to integrate with the power grid [29]. Bashash and Fathy designed a cost-optimal charging pattern for EVs that want to minimize the cost when charge in a time-varying pricing network [30]. Alesiani *et al.* focused on the routing problem of EVs when they want to decide the charging destination with consideration of the charging cost, remaining energy etc [31]. In addition, some works focus on new ideas. For example, providing mobile charging rather than charging at changeless places for EVs [32] or designing sustainable transportation rather than merging into the current one [33].

There is also some research on the placement of charging stations. Tan and Lin proposed to site the charging stations mainly concerning the demand flow and its uncertainty [34]. Unfortunately, their work fails to consider the interactive and implicitly competing EV drivers. Timpner and Wolf designed a scheduling strategy in the case EVs are charged in carparks [35]. However, this is not applicable to the general case for the potential large number of users in the city, because equipping each carpark with charging infrastructure is not realistic. Hausler *et al.*'s work also combines charging and parking [36]. Baouche *et al.* modeled the charging stations with a modified Fixed Charge Location Model mixed with a p-dispersion constraint, which is used to minimize the charging cost and construction cost [37]. Although accurate estimation of travel and energy demand was proposed, the work ignored the influence from the self-directed EV drivers' behavior.

Our work aims to propose a new angle of view in placing charging stations. Firstly, our model can merge the model with real-world data easily, such that we can provide practical solutions for concrete problem. Secondly, we fill the gap of previous works and model the influence of humans drivers in the charging system by using the game-theoretical framework to capture their selfish and strategic charging behavior.

VII. Conclusion

The key contributions of this paper include: (1) a realistic model for the CSPP in cities like Singapore considering the interactions among charging station placement, EV drivers' charging activities, traffic congestion and queuing time: (2) an equivalent single level CSPP of the bi-level CSPP optimization problem obtained through exploiting the structure of the charging game; (3) an effective heuristic approach that can speed up the mixed integer CSPP with a large amount of non-linear constraints; (4) theoretical analysis on PoA and corresponding experiments for the charging game; (5) experiments results based on real data from Singapore, which show that our approach solves an effective allocation of charging stations and outperforms baselines.

ACKNOWLEDGEMENTS

This article is an extension of work [38].

REFERENCES

[1] K. Schneider, C. Gerkensmeyer, M. Kintner-Meyer, and R. Fletcher, "Impact assessment of plug-in hybrid vehicles on pacific northwest distribution systems," in *Proc. IEEE Power Energy Soc. Gen. Meet. (PES)*, Jul. 2008, pp. 1–6.

[2] A. Hess, F. Malandrino, M. B. Reinhardt, C. Casetti, K. A. Hummel, and J. M. Barceló-Ordinas, "Optimal deployment of charging stations for electric vehicular networks," in *Proc. 1st Workshop Urban Netw. (UrbaNe)*, 2012, pp. 1–6.

[3] A. Y. S. Lam, Y.-W. Leung, and X. Chu, "Electric vehicle charging station placement," in *Proc. 4th IEEE Int. Conf. Smart Grid Commun. (SmartGridComm)*, Oct. 2013, pp. 510–515.

[4] A. Y. S. Lam, Y.-W. Leung, and X. Chu, "Electric vehicle charging station placement: Formulation, complexity, and solutions," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2846–2856, Nov. 2014.

[5] Z. Liu, F. Wen, and G. Ledwich, "Optimal planning of electric-vehicle charging stations in distribution systems," *IEEE Trans. Power Del.*, vol. 28, no. 1, pp. 102–110, Jan. 2013.

[6] T. D. Chen, K. M. Kockelman, and M. Khan, "The electric vehicle charging station location problem: a parking-based assignment method for Seattle," in *Proc. 92nd Annu. Meet. Transp. Res. Board (TRB)*, 2013, pp. 1213–1254.

[7] I. Frade, R. Anabela, G. Gonçalves, and A. Antunes, "Optimal location of charging stations for electric vehicles in a neighborhood in lisbon, portugal," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 2252, pp. 91–98, Dec. 2011.

[8] H. Wang, Q. Huang, C. Zhang, and A. Xia, "A novel approach for the layout of electric vehicle charging station," in *Proc. Int. Conf. Apperceiving Comput. Intell. Anal. (ICACIA)*, Dec. 2010, pp. 64–70.

[9] S. Funke, A. Nusser, and S. Storandt, "Placement of loading stations for electric vehicles: No detours necessary!" in *Proc. 28th AAAI Conf. Artif. Intell. (AAAI)*, 2014, pp. 633–658.

[10] F. He, D. Wu, Y. Yin, and Y. Guan, "Optimal deployment of public charging stations for plug-in hybrid electric vehicles," *Transp. Res. B, Methodol.*, vol. 47, pp. 87–101, Jan. 2013.

[11] M. Pierre, C. Jemelin, and N. Louvet, "Driving an electric vehicle. A sociological analysis on pioneer users," *Energy Efficiency*, vol. 4, no. 4, pp. 511–522, Nov. 2011.

[12] M. K. Hidrue, G. R. Parsons, W. Kempton, and M. P. Gardner, "Willingness to pay for electric vehicles and their attributes," *Resour. Energy Econ.*, vol. 33, no. 3, pp. 686–705, 2011.

[13] J. Gan, B. An, H. Wang, X. Sun, and Z. Shi, "Optimal pricing for improving efficiency of taxi systems," in *Proc. 23rd Int. Joint Conf. Artif. Intell. (IJCAI)*, 2013, pp. 2811–2818.

[14] J. Gan, B. An, H. Wang, X. Sun, and Z. Shi, "Optimal pricing for improving efficiency of taxi systems," in *Proc. IJCAI*, 2013, pp. 2811–2818.

[15] "Smart mobility 2030—Singapore ITS strategic plan," Land Transp. Authority Singapore, Singapore, Tech. Rep., 2014. [Online]. Available: https://www.lta.gov.sg/content/ltaweb/en/roads-and-motoring/managing-traffic-and-congestion/intelligent-transport-systems/SmartMobility2030.html

[16] S. B. Woo. (Mar. 2014). *LTA to Develop a Road Map for Electric Vehicles*. Accessed: Nov. 24, 2014. [Online]. Available: http://www.todayonline.com/singapore/lta-develop-road-map-electric-vehicles

[17] E. Mileage. (Nov. 2014). *All-Electric Vehicles (EVs)*. Accessed: Nov. 24, 2014. [Online]. Available: http://www.fueleconomy.gov/feg/evtech.shtml

[18] M. Boarnet, E. J. Kim, and E. Parkany, "Measuring traffic congestion," *Transp. Res. Rec., J. Transp. Res. Board*, vol. 1634, no. 1, pp. 93–99, 1998.

[19] R. Banner and A. Orda, "Multipath routing algorithms for congestion minimization," *IEEE/ACM Trans. Netw.*, vol. 15, no. 2, pp. 413–424, Apr. 2007.

[20] R. L. Bertini, "You are the traffic jam: An examination of congestion measures," *Proc. 85th Annu. Meet. Transp. Res. Board*, 2006, pp. 1–17.

[21] M. Sweet, "Does traffic congestion slow the economy?" *J. Planning Literature*, vol. 26, no. 4, pp. 391–404, 2011.

[22] C. Wang, M. Quddus, and S. Ison, "A spatio-temporal analysis of the impact of congestion on traffic safety on major roads in the UK," *Transportm. A, Transp. Sci.*, vol. 9, no. 2, pp. 124–148, 2013.

[23] N. Nisan, T. Roughgarden, E. Tardos, and V. V. Vazirani, *Algorithmic Game Theory*. Cambridge, U.K.: Cambridge Univ. Press, 2007.

[24] M. Tambe, A. X. Jiang, B. An, and M. Jain, "Computational game theory for security: Progress and challenges," in *Proc. AAAI Spring Symp. Appl. Comput. Game Theory*, 2014, pp. 1–6.

[25] E. Koutsoupias and C. Papadimitriou, "Worst-case equilibria," in *Proc. STACS*, 1999, pp. 404–413.

[26] N. Wang, Y. Liu, and G. Fu, "City readiness system assessment of electric vehicle charging infrastructure," in *Proc. IEEE 17th Int. Conf. Intell. Transp. Syst. (ITSC)*, . 2014, pp. 2805–2810.

[27] R. Faria, P. Moura, J. Delgado, and A. T. de Almeida, "Managing the charging of electrical vehicles: impacts on the electrical grid and on the environment," *IEEE Intell. Transp. Syst. Mag.*, vol. 6, no. 3, pp. 54–65, Jul. 2014.

[28] E. S. Rigas, S. D. Ramchurn, and N. Bassiliades, "Managing electric vehicles in the smart grid using artificial intelligence: A survey," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 4, pp. 1619–1635, Aug. 2015.

[29] R. J. Rei, F. J. Soares, P. M. R. Almeida, and J. A. P. Lopes, "Grid interactive charging control for plug-in electric vehicles," in *Proc. 13th Int. IEEE Conf. Intell. Transp. Syst. (ITSC)*, Sep. 2010, pp. 386–391.

[30] S. Bashash and H. K. Fathy, "Cost-optimal charging of plug-in hybrid electric vehicles under time-varying electricity price signals," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 5, pp. 1958–1968, Oct. 2014.

[31] F. Alesiani and N. Maslekar, "Optimization of charging stops for fleet of electric vehicles: A genetic approach," *IEEE Intell. Transp. Syst. Mag.*, vol. 6, no. 3, pp. 10–21, Jul. 2014.

[32] S. Huang, L. He, Y. Gu, K. Wood, and S. Benjaafar, "Design of a mobile charging service for electric vehicles in an urban environment," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 2, pp. 787–798, Apr. 2015.

[33] S. Mehar, S. Zeadally, G. Rémy, and S. M. Senouci, "Sustainable transportation management system for a fleet of electric vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 16, no. 3, pp. 1401–1414, Jun. 2015.

[34] J. Tan and W.-H. Lin, "A stochastic flow capturing location and allocation model for siting electric vehicle charging stations," in *Proc. IEEE 17th Int. Conf. Intell. Transp. Syst. (ITSC)*, Oct. 2014, pp. 2811–2816.

[35] J. Timpner and L. Wolf, "Design and evaluation of charging station scheduling strategies for electric vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 579–588, Apr. 2014.

[36] F. Hausler, E. Crisostomi, A. Schlote, I. Radusch, and R. Shorten, "Stochastic park-and-charge balancing for fully electric and plug-in hybrid vehicles," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 895–901, Apr. 2014.

[37] F. Baouche, R. Billot, R. Trigui, N.-E. El Faouzi, "Efficient allocation of electric vehicles charging stations: Optimization model and application to a dense urban network," *IEEE Intell. Transp. Syst. Mag.*, vol. 6, no. 3, pp. 33–43, Jul. 2014.

[38] Y. Xiong, J. Gan, B. An, C. Miao, and A. L. C. Bazzan, "Optimal electric vehicle charging station placement," in *Proc. 24th Int. Joint Conf. Artif. Intell. (IJCAI)*, 2015, pp. 2662–2668.

Yanhai Xiong received the B.S. degree from the University of Science and Technology of China. She is currently pursuing the Ph.D. degree with the Joint NTU-UBC Research Center of Excellence in Active Living for the Elderly (LILY), Interdisciplinary Graduate School, Nanyang Technological University. Her primary research interests include game theory and its application on intelligent interactive systems.

Jiarui Gan received the M.Eng. degree in software engineering from the University of Chinese Academy of Sciences, and the B.Sc. degree in geographic information science from East China Normal University. He is currently pursuing the D.Phil. (Ph.D.) degree with the Department of Computer Science, University of Oxford. His research lies in the areas of computational game theory and multiagent systems, with a particular focus on security games and policy optimization of transportation systems. Prior to enrolling at Oxford, he worked as a Research Associate with Nanyang Technological University.

Bo An received the Ph.D. degree in computer science from the University of Massachusetts, Amherst. He is currently a Nanyang Assistant Professor with the School of Computer Science and Engineering, Nanyang Technological University, Singapore. He has published over 70 referred papers at AAMAS, IJCAI, AAAI, ICAPS, KDD, JAAMAS, AIJ, and ACM/IEEE Transactions. His current research interests include artificial intelligence, multiagent systems, game theory, and optimization. He is a member of the Editorial Board of JAIR and the Associate Editor of JAAMAS. He was a recipient of the 2010 IFAAMAS Victor Lesser Distinguished Dissertation Award, an Operational Excellence Award from the Commander, First Coast Guard District of the United States, the Best Innovative Application Paper Award at AAMAS-12, the 2012 INFORMS Daniel H. Wagner Prize for Excellence in Operations Research Practice, and the Innovative Application Award at IAAI-16. He was invited to give Early Career Spotlight talk at IJCAI-17. He led the team HogRider, which won the 2017 Microsoft Collaborative AI Challenge. He was elected to the board of directors of IFAAMAS.

Chunyan Miao is currently a Professor with the School of Computer Science and Engineering, Nanyang Technological University. She is also the Director of the NTU-UBC Joint Research Centre of Excellence in Active Living for the Elderly (LILY). Prior to joining NTU, she was an Instructor and Post-Doctoral Fellow with the School of Computing, Simon Fraser University, Canada. Her research focus is on studying the cognitive and social characteristics of intelligent agents in multi-agent and distributed AI/CI systems, such as trust, emotions, motivated learning, and ecological and organizational behavior. She has made significant contributions in the integration of the above research into emerging technologies, such as interactive digital media, including virtual world, social networks, and massively multi-player game; cloud computing; mobile communication; and humanoid robots.

Ana L. C. Bazzan received the Ph.D. degree from the University of Karlsruhe, Germany. She is a Full Professor with the Computer Science Institute, UFRGS, Porto Alegre, Brazil. Her main research interests are multiagent systems, multiagent learning, complex systems, machine learning, agent-based simulation, and applications of AI and multiagent techniques in traffic simulation and control. Her professional activities include: a former Associate Editor of the journals *Autonomous Agents and Multiagent Systems*, *Advances in Complex Systems*, and *Journal of Multiagent and Grid Systems*; currently in the Editorial Board of the *Journal on Vehicle Routing Algorithms*; a Co-General Chair of the AAMAS 2014 Conference; served as an Area Chair, as a Senior Member, and as a Reviewer in various conferences in the area of multiagent systems.